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Terence Johny

Solutions Engineering Specialist

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Terence.johny@emc.com

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Ψ. The Preamble

"Dimidium facti qui coepit habet; sapere aude; incipe!"

"He who has begun has half done. Dare to be wise; begin!"

— *Horace, 20 BC, (Epistles. I. II, 40)*

Quarks have been making waves lately… Randomness and Uncertainty rule. And the duel of *"subatomic-duality"* has been an all-consuming quantum quest. The collapse of the wave function **(Ψ)** sounds like a sci-fi thriller unfolding at the nanoscale in femtoseconds, and we are now entering the realm of quantum mechanics where classical laws of nature fail and events play out at a subnuclear scale, with inexplicable universal synchronicity that spans across *space and time*.

A few decades ago, this may have been dismissed as phantasms born of our fevered imagination, but the pace at which quantum mechanics has progressed over the last few decades is jolting us into a passionate quest to solve the meaning of reality itself. The effort is real, the progress is pronounced, and the revolution is coming. And like all good revolutions, at heart lies a very simplistic idea or a question rather…*What's the state of the cat?*

Yes, the fate of Schrodinger's fabulous feline is unknown *(until you peek),* but the world has been steadily pacing toward the goal of Quantum supremacy. In the past, other emerging technologies may have been dubbed as **"paradigm shifts,"** only to succumb to being victims of a *hyperbole*. But, in the instance of quantum computing, this would be a *gross* understatement.

In this paper, we will explore the quantum world, trying to decipher some of the core phenomena that govern the quantum realms, *ever so briefly,* while trying to bind those concepts to modern quantum computing architecture. We will look at the basics of quantum computing, its classical brethren, and the problems it's trying to solve. We explore seriatim, the evolution of quantum devices, and examine its radical effects on *AI, Machine Learning,* and be as bold to define a new type of internet for the quantum world. **From,** optimizing radiation to kill cancerous cells, raking in quantum profits for investors by financial modeling, weather forecasting, and cryptography **to,** the detection of traffic jams *"45 minutes before they even occur",* the list of potential quantum applications has been unraveling in real-time, and the promises seem **endless**. But, in this euphoria for the race to supremacy, we mustn't hesitate to debate the morality of "quantum progress." Will it improve the quality of the human life experience, or will it add another layer of complexity to our existence? Will this help us as a species or divide us further into camps of cult loyalties.

Our sanguinity for progress must not overshadow the collective human endeavor that set us on this path of self-discovery in the first place, *that is*, to answer the fundamental underlying questions of our existence and our exploration of what we express as our *space-time reality.*

Σ. Genesis 1:3 - "Let there be light," and light appeared.

> "*There is nothing new to be discovered in physics now. All that remains is more and more precise measurement.* —
>
> *Lord Kelvin, 1900*

The invention of the electric light bulb was a pivotal moment in modern history. It accelerated the industrial revolution, jumpstarted new enterprises, and catalyzed innovations in energy transmission, electric motors, and even home appliances. ***Talk about a bright idea***!

While the invention itself has many appellants, it was Edison's ingenuity that made him the first commercial success. Edison learned from the successes and failures of his counterparts that the critical problem was the filament. An electric current runs through a hair-like thread that has enough resistance to generate heat energy, which spits out—you got it **"Light."**
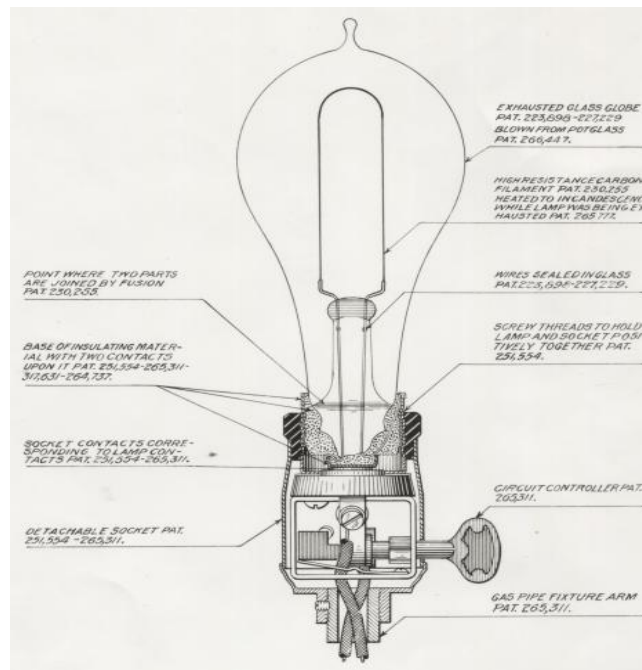
Figure 1: Edison's patent with the signature 'screw-holder' assembly.

As Edison installed his very first carbonized *bamboo-fiber-filament bulbs* at 449 Water Street, New York City in 1881, they were capable of lasting only for about 1200 hours. This average would rapidly improve over the years to 10,000 hours*; Courtesy: ductile tungsten, inert gases, and assiduous scientific pursuit.* Nonetheless, there was another storm brewing across the Atlantic— intertwined with the aforementioned *"filament"* and its **whitish-yellow glow**, that was upsetting the scientific *status-quo* of the time.

The Ultraviolet Catastrophe . . .

While the *"glow"* from the *light bulb* was illuminating parts of the world, it was simultaneously pushing *Newtonian* physicists into a dark alley of introspection. They began to notice a strange pattern emerging through the **"thermal radiation,"** *i.e.* the color of the light emitted from the "blackbody." A blackbody is simply any object that absorbs all radiation incident on it, like the filament in a light bulb or the celestial sun.

If we heat a piece of metal, it will begin to glow, first, turning reddish and then getting more and more yellowish-white as the temperature increases. In physics terms, it emits electromagnetic radiation. Understanding this phenomenon theoretically and being able to precisely predict the spectrum of radiation emitted from objects and gases was one of the hottest topics *(no pun intended)* of physics in the late 19th century. The consensus of the time was that the thermal radiation from a blackbody was characteristic of its temperature. Among the prevailing theories that attempted to explain the radiation spectrum of a blackbody, the ***Rayleigh-Jeans law*** was able to predict the radiation spectrum for a subset of frequencies of light and mathematically expressed their findings as follows.

$$u_\nu = \frac{8\pi\nu^2}{c^3}kT$$

where, u_ν is the energy density per unit frequency interval, k is the Boltzmann's constant, T is the absolute temperature of the radiating body, and **c** is the speed of light in a vacuum. Although the equation worked for a subset of frequencies, it diverged drastically with experimental data, as shown in *figure 2.* The Rayleigh-Jeans law plotted its way straight into infinity for the intensity of light as its wavelength inched toward smaller values. This result meant that the universe contained an infinite amount of high energy radiations, which would make human life impossible. This failure was infamously dubbed as the ***"Ultraviolet Catastrophe"*** because it alluded to the fact that there was something wrong with the classical notions of physics that went into formulating the *Rayleigh-Jean* law which held for a subset of conditions but failed catastrophically for another set of equally valid terms.

Mr. Planck to the rescue and the serendipitous birth of "Quanta."

In 1900, Max Planck had turned his attention to the *Ultraviolet Catastrophe* with the hope that by understanding the fundamental relationship between the intensity *vs.* frequency of light, he would be able to develop a more efficient light bulb. Planck resorted to a mathematical approximation in a desperate attempt to explain away the ultraviolet catastrophe and ended up making a phenomenal assumption that, albeit fortuitously, would change the course of science forever. In a stark departure from his own beliefs and quite uncharacteristically, Planck postulated that that energy was not continuous, as stated by classical mechanics; rather, it was packetized and took specific energy levels. And any other values, including the ones in between the allowed levels, were impossible. He determined that, for an atom oscillating with a frequency ν, the allowed energy levels were integer multiples of the base energy unit $E = h\nu$ where the Planck's constant that was later experimentally calculated to be h = 6.62607004x 10[-34] Js[1]
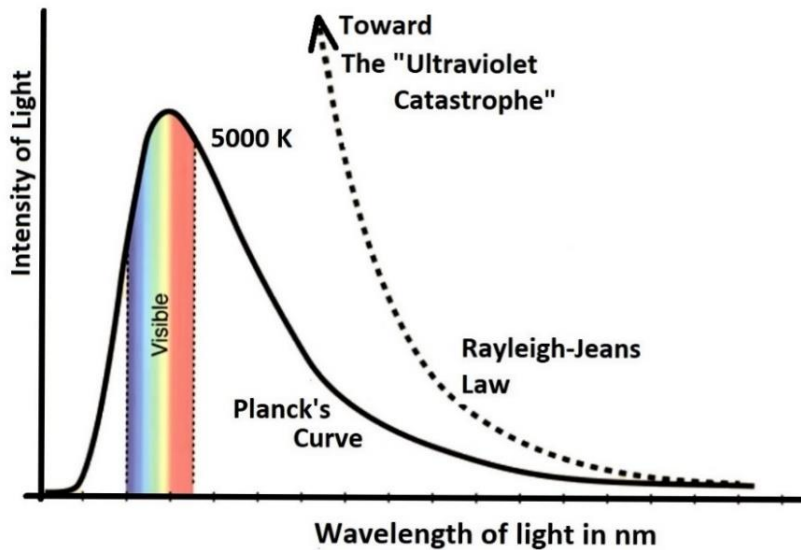
Figure 2: Depicting the difference between the Rayleigh-Jeans Curve and Planck's Curve.

The packets of energy were called *"quanta",* and Planck's clever mathematical trickery had allowed him to equipartition energy in a way that solved the ultraviolet catastrophe by fitting the observed radiation curve exactly to the experimental results. Applying this new approach described the blackbody spectrum accurately, across all wavelengths of light, and while this was a triumph for the radiation spectrum problem, the implications of Planck's quanta were about to revolutionize physics. Planck had taken the first *"quantum leap,"* but it would still take an *"Einstein"* to appreciate the significance of *"h"* that would soon ascend to become the **"universal constant of quantization."**

The Photoelectric Dilemma & Quantum Discontinuity

If scientists are sure about anything related to light toward the end of the 19th century, then it is about the wave nature of light. Light is a wave, and this view has been long-established by experimental observations of optical phenomena like diffraction, interference, polarization, reflection, and refraction, all pointing toward a *"wavy light."*

Into this "*wavy-light"* world Philipp Lenard, an assistant of Heinrich Hertz, shone a beam of light on a metal surface. Now, under favorable conditions, light when incident on metal will transfer enough energy that can free electrons from a surface of the solid. The emitted electrons are called **"photoelectrons."** The *dilemma*, Lenard observed stem from the fact that the intensity of light did not affect the energy of the photoelectrons, specifically their kinetic energy. Very bright, intense light, and dim light had the same effect, and this was utterly unanticipated. As per established classical laws, the energy of light was directly related to its intensity, and the brighter the light, the faster the electrons would be ejected from the metal surface. This assumption, however, was not the case.
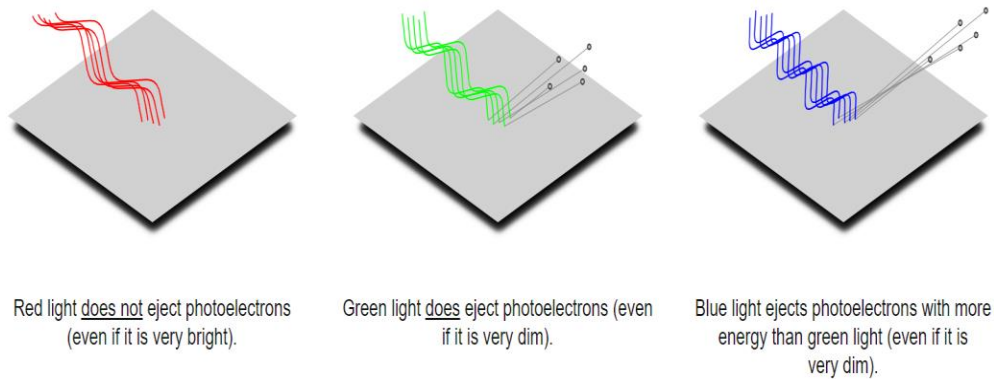
Red light <u>does not</u> eject photoelectrons (even if it is very bright).

Green light <u>does</u> eject photoelectrons (even if it is very dim).

Blue light ejects photoelectrons with more energy than green light (even if it is very dim).

Figure 3: Light waves on a metal surface. Higher the light frequency not "intensity", faster the photoelectrons escaped [2]

If we were to imagine light as waves in the ocean and electrons as buoys tethered to an anchor, then it's easy to relate to how the weak waves have little or no effect on the buoys, but strong waves could displace a buoy from it anchor and set it free. Weak waves and strong waves are analogous to the intensity of the wave. Stronger the intensity, the more the displacement of the buoys, similar to what we would expect with light, only that it's not what happens. In fact, the photoelectric effects describe an ocean where a tsunami wave **wouldn't** tip over a canoe, but a tiny ripple could fling ships into the air. Something did not add up.

The photoelectric effect described the energy transference from light to electron. A photon with energy E_p collides with an electron on a metal surface and transfers energy, a portion of which goes toward releasing the electron from its anchor—the nucleus of the metal atoms and the rest is used as kinetic energy. Using this concept, if we increased the intensity of light *(stronger waves),* then the electron should have more kinetic energy after it has escaped the metal surface.

Nevertheless, the theory contradicted the experiment. *Enter Einstein,* who solved this dilemma using a page from Planck's "*quanta of energy.*" Einstein proposed that the energy of the photoelectrons increases linearly with the frequency of the light and not its intensity; He hypothesized that that like energy, light too, came in packets or *quanta*; called **photon** particles and it was the energy contained in each photon particle *(frequency)* rather than the number of packets *(intensity)* that dictated the kinetic energy of the photoelectrons, expressed mathematically as,

$$K_{max} = h(f - f')$$

The photoelectric effect brought the emerging concept of the **wave-particle duality** of light under the spotlight. Light exhibits both wave and particle-like characteristics, each manifesting according to the circumstances, and this perplexed a lot of physicists. The duality of light became a subject of much-heated debate. But what no one foresaw at the time was that the *duality of light* was just an initiation to a more radical idea, the *"duality of matter"* itself.

. . .

Φ. The Ghost in the Particle
Quantum Weirdness and Spooky Action

" Those who are not shocked when they first come across quantum theory cannot possibly have understood it. "

- *Niels Bohr, 1971 (Physics and Beyond, p.206)*

The beauty of an experiment lies in its simplicity and elegance of thought. The double-slit experiment is one such stunning example that puts to test our fundamental understanding of nature and reality. Let me attempt to illustrate how.

Reverting to the light bulb, suppose we use it as a source of light shining against a metal sheet with two slits. The slits are narrow and at a defined distance to each other. As light squeezes through the slits, it diffracts, and the waves that come out the other end overlap and interfere, creating an *"interference pattern"* on the screen, as depicted in *figure 4*. The interference pattern is a series of consecutive bright and dark bands. When the crest of two or more waves interfere, they add up, creating a constructive interference as opposed to when the troughs of two waves cancel out, creating a destructive interference that manifests as bright and dark bands on the screen.
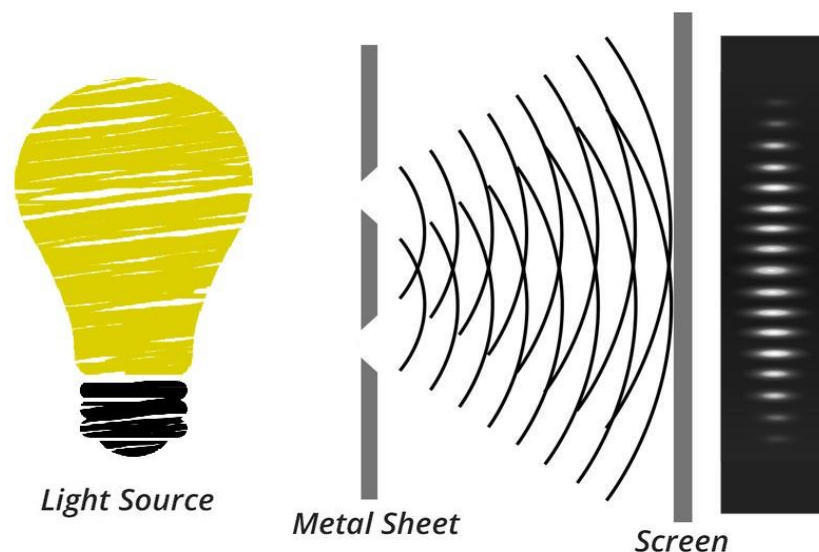

Figure 4: Wave Interference Pattern, as seen in Young's Experiment in 1800 A.D.

By the late 19th century, the argument that light is both a wave and a particle has caught on. And the *"quantization of light"* coupled with the wave-like behavior is a topic of continued debate. The belief at the time was that depending on the type of experiment; light would exhibit wave or particle-like properties. **The double-slit experiment with the light bulb highlights the *"wavy"* nature of light, and everything seems to be in order**. Hold on to that thought as we go further down this rabbit-hole.

What if we repeated the double-slit experiment with marbles instead of light. What would we expect to see? Suppose we have at our disposal a gun that fires marbles at the metal sheet with the double slits. As you may have guessed, some of the marbles would go through the upper slit(1) some through the

lower slit(2), and some would be deflected and not pass at all. If every marble imprints its point of contact on the screen, the pattern we'd get on the screen would be two straight lines. We will refer to this as a **"particle pattern."** This pattern is what we would come to expect of any particles, even if we tried an iteration of this experiment with sand particles, we would get a *particle pattern*. This understanding leads us to a natural conclusion about the double-slit experiment; waves through the slits will create an *"interference pattern"* and particles will create a *"particle pattern"* and this should hold for all particles of matter and all waves of nature.

An electron is an elementary particle and has a mass of *9.10938356 × 10^{-31}* kilograms. If we use an electron gun and fire electrons instead of marbles, we should get a *"particle pattern,"* right? To be as precise as possible, since we are dealing with such microscopic particles, let's do this methodically and begin with one slit covered, load our electron gun and fire. Lo and behold, as expected, we get the particle pattern with a single stripe, as seen in *figure 5*.
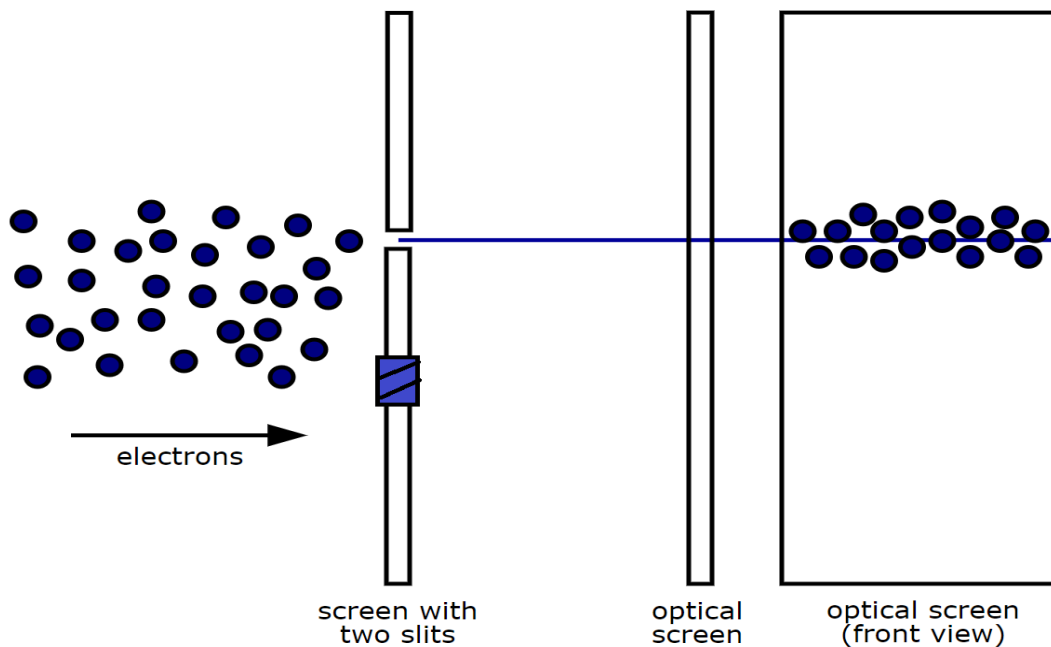

Figure 5: The particle pattern of electrons through the double-slit with slit 2 covered.

Well, so far, so good, however, I promised you weirdness, and I am about to deliver. The mystery begins when we open the second slit and observe the pattern on the screen, as depicted in *figure 6*. When a stream of electrons is fired at the sheet with a double slit, we get a pattern that can only be interpreted as a wave-like *interference pattern* and not the *particle pattern* that all types of particle in nature exhibit. Certainly, we did see the electrons exhibit *particle pattern* a moment ago when we had slit(2)covered in the previous run. Even when the experiment is carefully calibrated so that at any given point in time only one electron is fired at a time with a discernable interval between, we observe the same wave-like interference pattern, although at first, we can trace each particle randomly landing on the screen in a localized manner at specific points, gradually over time as more and more electrons are fired, we always end up with the wave interference pattern build-up.
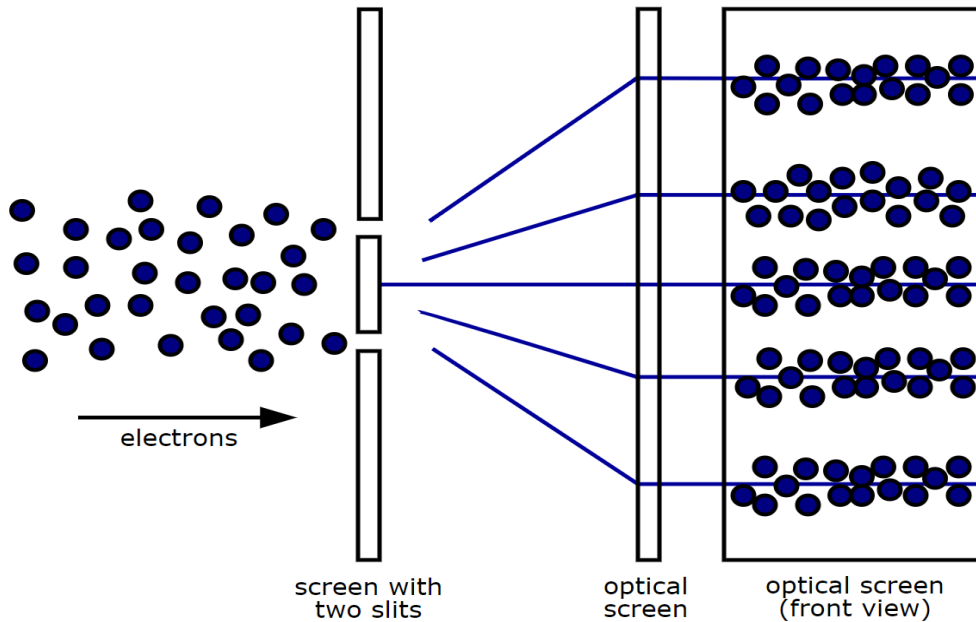
Figure 6: The double-slit electron interference pattern. Similar to what we would expect of waves.

Take into consideration the fact that electrons are one of the elementary particles that are present in everything in the universe. They are particles that make up matter, the marbles we just used for our experiment are made up of millions of electrons. Nevertheless, one has to contend that these particles are behaving as waves? Let's put on our engineering caps and investigate this a bit further. We will build ourselves an *"electron detector"* and place this device near the upper slit. When an electron passes through the upper slit, the detector will beep and keep count of all the electrons passing through the upper slit. So, we fire up our experiment, and the resultant pattern on the screen that we end up with is depicted in *figure 7*.

While the resulting particle pattern is a relief because it reaffirms our belief that electrons are particles, at the same time, on the other hand, that's a different pattern from the wave interference pattern we saw before we added the detector in place. Nothing else has changed other than a detector keeping track of the electrons. Given these facts, it does seem to indicate that the electrons are somehow aware of the detector. Even spookier is the inference that leads us to believe that the electrons are collectively mindful that they are being observed and are coordinating among themselves as to what pattern results on the screen; is that even possible?

Well, let's try a neat little trick; let's leave the detector as is, but turn it off and *rinse and repeat*. If the electrons are changing their behavior just because they are somehow sensing the presence of the detector, we should get the same *particle pattern* as we did when it was switched on. So, once again, we fire up our experiment, and this time, we switch off the detector leaving it in place above the upper slit and wait and watch. The resultant pattern on the screen that we end up with is depicted in *figure 8*. By some means, the electrons are back to producing a wave-like interference pattern. What's weirder is that somehow again, there seems like there are some collective coordination and awareness. Surely no *"matter"* behaves like this.
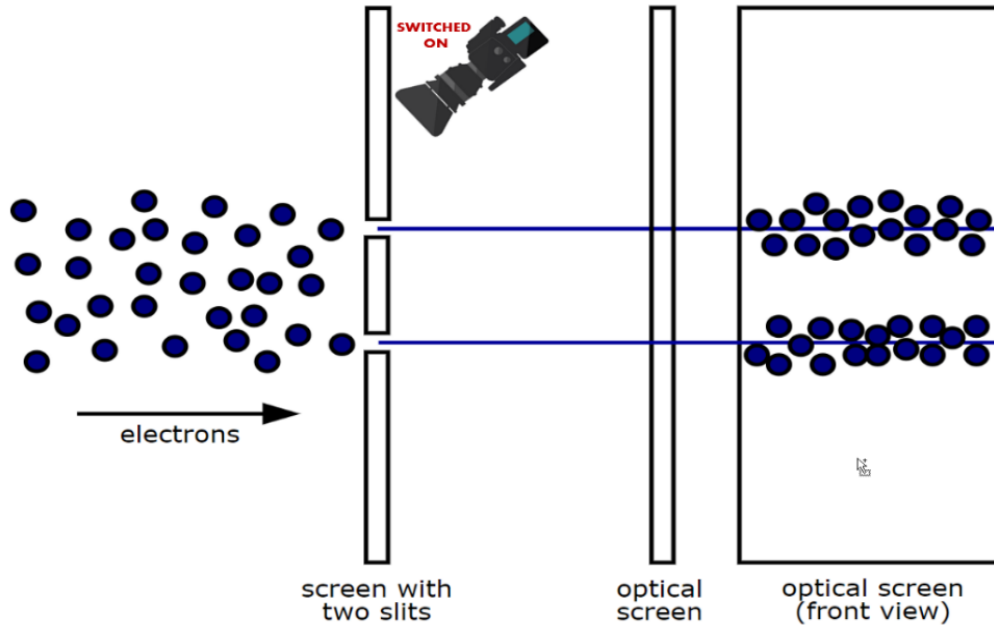
Figure 7: The particle pattern re-emerges when we place a detector to track the electrons
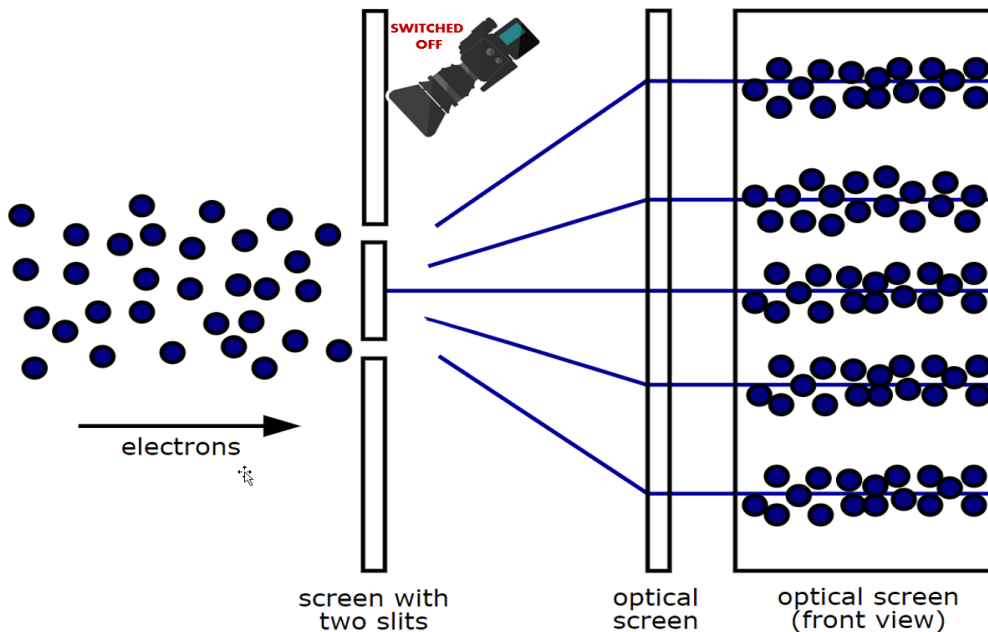
Figure 8: The wave interference pattern re-emerges with the detector switched off.

At this point, I feel obliged to clarify that none of this is science fiction. Neither is this some *"Gedanken-experiments*[6]*",* the kind theoretical physicists conjure up for fun. Each of these experiments and their variations has been carried out by several teams of scientists around the world.

In 1927, *Davisson and Germer* demonstrated the wave-particle duality by diffracting electron beams through a nickel crystal. In 1989 Akira Tonomura and co-workers at Hitachi conducted the double-slit experiment with single electrons at any one time in the apparatus when they observed the buildup of

the wave interference pattern[3]. Similar results were obtained with other subatomic particles as well viz. neutrons, atoms, and even large molecules like the *Buckminsterfullerene*, aka- **"Buckyballs"-** a sizeable spherical molecule of 60-Carbon atoms.

Let us start by considering the facts of the double-slit experiment and its findings. We know the electrons' position at the beginning and the end. We also see the individual electrons always register on the screen at a localized point, just like any other macroscopic particle in nature. However, the eventual wave-like pattern build-up requires us to deliberate that the electron *"en route"* is traveling like a wave. If that were true, it would imply that the electron passes through both slits at the same time then interferes with itself, and just before it's about to hit the screen collapses back into a particle. What causes the collapse and how do we explain the even weirder observation that all the electron particles seem to be interacting with each other to create the wave-like interference pattern. Also, how is this behavior affected by the detector? Is matter exhibiting the same wave-particle duality as light?
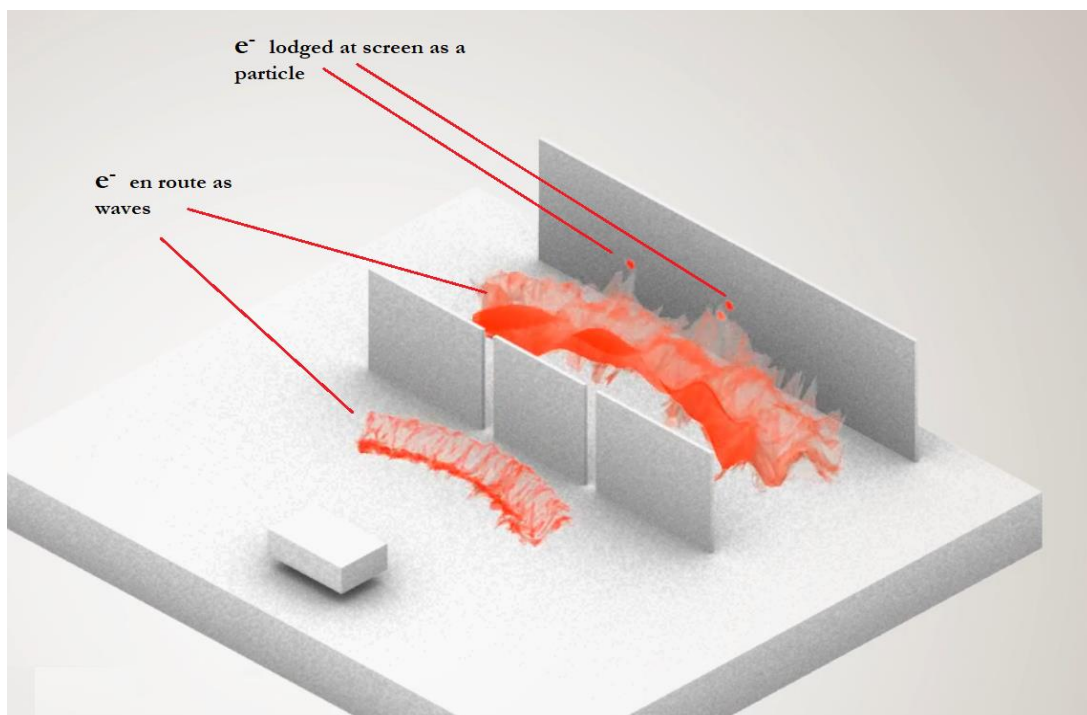
Figure 9: An animation snapshot depicting the double-slit experiment wave-particle duality. [4]

Hypotheses, Interpretations & the Wave Function

Decoding this mystery has been one of the central themes of Quantum Mechanics. Among the various interpretations and inferences' around the duality of light and matter, Niels Bohr and his followers left us with a formidable legacy – "*The Copenhagen Interpretation."* [8]

The story and evolution of Bohr's legacy are lengthy and complicated, but at the core was the idea of "*Complementarity"-* that the *wave-particle* duality did not manifest simultaneously, but the electron or photon or any other subatomic particle either manifested as a wave or a particle depending on the circumstances. The *wave-particle* duality of matter and light is regarded as complementary facets of a single reality, as the two sides of a coin. The electron *(and by extension all "matter-waves")* sometimes

behave like waves and sometimes like particles, but never both together. Just as a coin tossed in the air, may either fall heads-up or tails, but never both at once. A very crucial comprehension of the interpretation is that the wave nature of light or matter did not refer to real physical waves like sound or water waves. Instead, it is comprised of waves of pure possibility. The mathematical model developed to describe this *wave of probability-distribution* is called the **"wave function."**

The wave function is interpreted as a probability-amplitude, where the square of the magnitude of the wave function describes the probability of an electron existing in a particular location famously known as the **Schrodinger Equation**:

$$H(t)\left|\Psi(t)\right\rangle = i\hbar\frac{\partial}{\partial t}\left|\Psi(t)\right. \quad \text{where;}$$

H is the Hamiltonian operator; it describes all the interactions that are affecting the state of the system, also known as the total energy of a particle. **Ψ** represents the wave function.

As proposed by Max Born in the 1920s, the waves are *'measures'* of probability. Probability waves related to the *Uncertainty* principle, leading to the idea that there is no deterministic reality of the *wave-particles* and identical electrons in identical circumstances, may behave differently, and the best we could do is statistically predict the most probable outcome. "Complementarity, uncertainty, and the statistical interpretation of Schrodinger's wave function were all related, and together they shaped a logical explanation of the physical meaning of quantum mechanics known as the *Copenhagen interpretation.*" [5]

In terms of the double-slit experiment, what the Copenhagen Interpretation was saying was that the electron traversing the double-slit exists only as a wave of probability that holds information about all possible paths and positions the particle can take to its destination. It suggests that the electron exists in all places at all times, occupying all states like a wave of energy and only materializes as a particle on measurement with a macroscopic system viz. detector or the interference screen itself. This transition of the quantum wave function to a classical deterministic state is popularly termed as the **"collapse of the wave function."**

This, however, is not the only interpretation that fits empirical and experimental observational evidence. In the mid-1950's Dr. Hugh Everett, III, revisited the collapse of the wave function postulate in his Ph.D. thesis only to emerge with his idea called the "relative-state metatheory" or more popularly known as the **"Many-Worlds Hypothesis."**

Here, unlike the Copenhagen Interpretation, observation, or measurement, does not collapse the wave function into a single reality; instead, it splits reality into multiple outcomes, each outcome a probability defined by the wave function itself. According to the many-worlds hypothesis, every possible result that can occur does occur. All outcomes exist simultaneously without collapsing or interfering with each other. All the possibilities that can exist do exist and are evolving in parallel universes, split into mutually unobservable but equally real worlds. Again, not science fiction but a real hypothesis that the mathematics supports.

The mystery surrounding the wave-function and whether it collapses into a single reality or evolves into multiple parallel realities or if there even is an objective reality is the *holy grail* of quantum mechanics.

Ω. Uncertainty is a Feature and Randomness a Function.

"Not only does God play dice, but He sometimes confuses us by throwing them where they can't be seen."

-   *Stephen Hawking, 1999* (L. *Does God Play Dice*)

As you gently tap on a tile at one of the ends of a row of dominoes, you witness it crash into its neighbor, which in turn crashes into its adjacent tile creating a ripple effect that continues until all the dominoes have toppled. "***Causality***" is a deeply rooted concept of science and an everyday occurrence of our macroscopic lives. The last domino tile falls *only* because you initiated the first tile. There are clear *cause and effects* that yields a very deterministic view of past and future events. Extrapolating causality to Newtonian mechanics meant that given knowledge of an object's current state of motion, predicting its trajectory of motion could be precisely determined. Well, not if Heisenberg had any say in this.

Heisenberg opposed this idea that an effect follows a cause, and determinate causality was inherent in all of nature. He contended that there are limits to the precision with which specific properties could be measured in quantum mechanics. In its purest form, the **Uncertainty Principle** [5] states that the *position and velocity* of an object cannot both be measured precisely, even in theory, simultaneously. He expressed his principle mathematically as;

$$\Delta p \Delta x \geq \frac{h}{4\pi}$$

where, $\boldsymbol{\Delta}$ is the uncertainty, and $\boldsymbol{h}$ is Planck's constant. $\boldsymbol{\Delta x}$ marks the pot("position"), and $\boldsymbol{\Delta p}$ is the momentum(*p=mass x velocity*). Note that the principle doesn't state that "e*verything is uncertain*," instead, it specifies the limits of certainty when we make measurements. Now, citing the *uncertainty principle* to the cop who stopped you for speeding at 45 miles/hr in a school-zone won't work. He knows your position and speed, hence *"the ticket."* But that doesn't mean the uncertainty does not exist. If we compared the mass of the moving object (*vehicle* **+** you) to the $\boldsymbol{h}$ = 6.62607004x 10^{-34} Js[1]**,** it's evident that the mass is so large that the uncertainty is negligible. While it can't get you off-the-hook with the speeding ticket, Heisenberg's *Uncertainty Principle* is ubiquitous at the sub-atomic scale and since the mass of subatomic protons, neutrons, electrons are in the **~10^{-30} kg** ranges the uncertainty is no longer negligible.

This uncertainty has nothing to do with any physical limitation of the measuring apparatus or the methodology, but it's a characteristic of nature's intimate connection to its quantum constituents. For subatomic particles like electrons, the more precise the measurement of the position, the more uncertain the momentum would be and vice versa. This was characteristic of any conjugate variable like *position and momentum* or *energy and time*. Classical physics was already contending with the ramifications of the *wave-particle duality* phenomenon when it was dealt with the heavy blow from the uncertainty principle.

Quantum Superposition and the curious case of the Cat in the Box.

For a very long time, we have taken for granted our objective reality with the universal acceptance of certain facts. The quantum world, however, denies the existence of any such accurate and unambiguous reality. From this arose a set of perplexing implications. The notion that nature is not deterministic but rather probabilistic with a *sprinkle of uncertainty* has led to incredible scientific and philosophical implications.

Schrodinger even devised a paradox attempting to highlight the absurdity of it all. He put a fictional cat in a box with a vile of poison and a single radioactive atom with the potential to decay at any moment. Inside was also a device that could detect if any radiation occurred, triggering a mechanism that would break the vile, killing the feline. Now, close the box or in *science-speak* "**isolate this system from observation**." According to the *Copenhagen Interpretation*, the radioactive atom takes all possible states which means it's decaying and not decaying at the same time and because of the mechanism in the box that also would say that the cat is in a state of "*dead and alive*" both at the same time. Both possibilities exist together in a **"Superposition"** of dead and alive states. Only if you open the box and observe, you force the collapse of all the possibilities into a single deterministic state. This superposition of states is one of the fundamental principles of quantum mechanics.
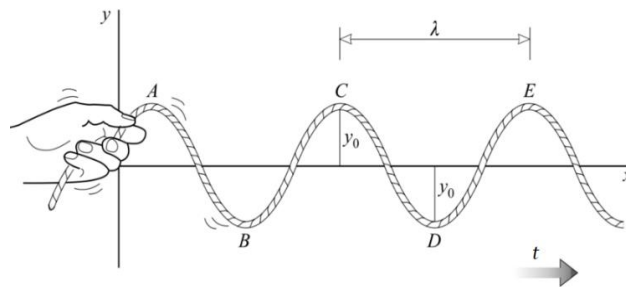

Figure 10: A wave function describes the shape of this wave in time t created by a vibrating string.

Quantum Superposition is referred to as a state "in-between" states. If we spin a coin on a surface, then while it spins, we cannot say it is either *heads* or *tails*. The coin exists in a superposition of heads and tails, and only when it falls flat out does it take a defined state of either *heads* or *tails*. Quantum particle superpositions can be imagined similarly. Although it's not the complete picture, it provides an intuitive basis, being very counter-intuitive to our everyday experience.

More formally, superposition can be explained by the help of wave functions. Think of an ordinary wave, such as depicted in *figure 10,* traveling down a piece of string. Using fundamental physics, we can formulate a ***wave equation***, which describes how the wave changes over space and time. A solution to the wave equation is called a ***wave function***, which represents the shape of the wave at every point in time ***t***. Armed with this knowledge, let's attempt a more scientific understanding of superposition.

For this, we will make use of a home-grown *Gedanken Experiment* [6], let us christen it the ***"Double Slit Virtual Box"*** experiment. Using similar conditions as in the double-slit experiment earlier, imagine placing a *"virtual box"* in the space between the electron-gun and the screen. The virtual box has a

unique property; in that, it lets particles in but never lets them escape, *i.e.* it holds the electron in, never allowing it to reach the screen ever. Thus, never letting the collapse of the wave function and containing the electron in a superposition of all possible positions within the virtual box.

Since we know that the electron behaves like a wave inside this virtual box space, similar to the standing wave in *figure 10*, we can apply the Schrodinger wave equation to calculate the wave function(solution) for an electron wave. We also said that the electron takes all possible positions within the virtual box. So, if we calculate the wave equation for each point **x** the particle takes in the virtual-box for time **t** of its journey, we could determine the probability amplitude of finding the electron at a point **x** at time **t**.

Following this logic, we derive multiple wave functions for each position x in the virtual box. So, for x_1, we have a solution in time **t**, and for x_2, we have another solution. And here's the thing: if we add the two wave functions for x_1 and x_2, we get the third solution for another point x_3 in time **t**. This result seems to indicate that the probability of finding the electron at point x_1 and the probability of finding it at x_2 when added up would give a third probability of finding it at a third position x_3 in the box. And if we go on adding multiple probabilities, we get new probabilities for a different place "x_n" which would mean that there are numerous probabilities of finding the electron at different locations within the virtual box at the same time. It can be safely assumed that the particle is in several places at once in a quantum superposition of all its states. Quantum Superposition is one of the quintessential principles of quantum mechanics and is the primary phenomenon that will be used for building quantum computers.

Quantum Entanglement's Spooky Action at a Distance

Indulge me, please, for a moment. Assume I have two identical coins A and B, and I spin coin A on earth, and while its spinning teleport myself to planet Vulcan *(you guessed it, my favorite vacation spot)* where I spin coin B. Under the laws of locality (*an object can be effected only by its surroundings)*, if I force coin B to fall tails up, that should and would have **no** impact on whether coin A on earth falls heads or tails up, correct? Well, experiments have shown that reality is much, much weirder than that. According to quantum mechanics, in addition to being in a superposition of states, quantum particles can be entangled with each other even if they are at the opposite ends of the universe. So, if coins A and B were quantum coins, then forcing coin B on planet Vulcan to "*tails up"* would instantaneously cause coin A on earth to fall "*heads up"* and vice versa. This phenomenon is known as quantum entanglement.

Einstein famously ridiculed this as spooky action at a distance and refused to accept regarding it as impossible because it meant that there was faster than the speed of light communication between the two quantum coins – the bedrock of his *theory of relativity*. He has since been proven wrong about quantum entanglement multiple times. *How do you think I teleported to Vulcan in the first place….* Quantum entanglement occurs when two particles become indissolubly linked.

At the University of Glasgow in Scotland, physicists have to their credit the first photo of quantum entanglement depicted in *figure 11.* The photo shows the entanglement of two photons. The physicists split the entangled photons up by passing them through a crystal and then pass one photon from each pair of entangled photons through a liquid crystal material known as **β**-barium borate, triggering four-phase transitions.

In the experimental setup [10], the entangled photons are sent from the bottom left, one half to the left, the rest through the phase filters. The phase filter changes the phase of the photon while the other photon of the entangled pair is sent straight ahead, avoiding the phase filters. But yet they undergo the same phase changes as their pair undergoes through the phase filters. The camera captured images of these at the same time, showing that they'd both shifted the same way despite being split. In other words, they were *entangled*.
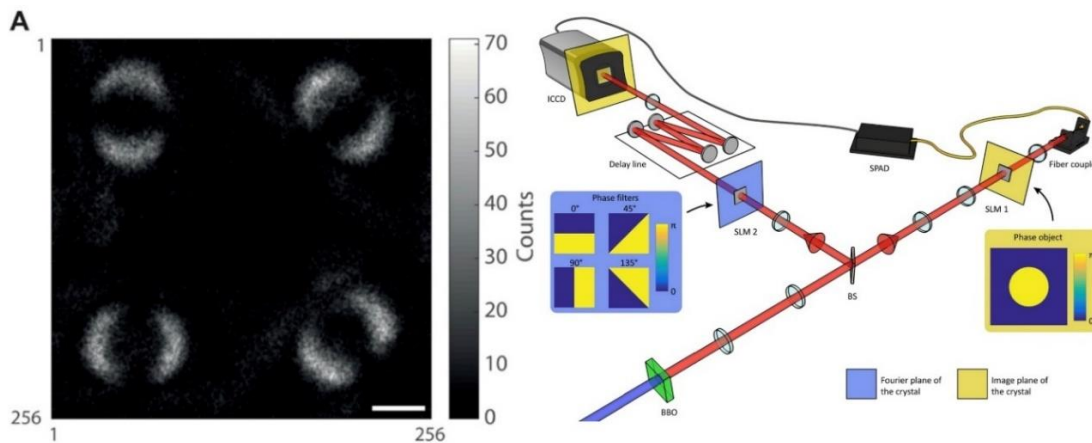

Figure 11: First-ever photo of quantum entanglement(left) with the experimental setup(right).(Moreau et al., Science Advances,2019)

Quantum Mechanics, huh, yeah… What is it good for? Absolutely everything [11]

Quantum Mechanics is admittedly a tough subject with intricate mathematics and scientific theory. The outsiders who brave the quantum world find themselves in confusing places with perplexing physics. It's not uncommon to come across declarations questioning its practical applications, and there's an ironic absurdity to seeing some of these comments on the internet, a global life-changing practical application of quantum mechanics. Without understanding the wave nature of electrons, it would be impossible to leverage the conductivity in silicon to produce transistors efficiently. The entire information technology industries rely on understanding quantum principles.

The contribution of Heisenberg, Schrödinger, and their followers not only has given us a unique perception into the very nature of reality itself but has also opened up new avenues of scientific progress. Everything from the light bulbs, lasers, nuclear reactors to ultra-precise atomic clocks capable of accuracy to a billion years are the result of years of devout effort to understand the quantum principles that govern the physics of these tools. Veritably, it wouldn't be hyperbole to state that "*Life*" itself is a quantum gift.

Life on earth would have never come to exist hadn't it been for our seemingly infinite source of energy – the Sun. And this *burning ball of fire* wouldn't be burning hadn't it been for nuclear fusion – enabled by a quantum phenomenon that lets subatomic particles take a *quantum walk through a wall.*

Quantum Tunneling and Walking through Walls

Nuclear fusion has reduced the Sun's mass by a total of 0.03% of its starting value since it came into being[12]. A small fraction on paper, but to put this into perspective, this loss is equivalent to the entire mass of Saturn. And only 5% of the stars in the known universe get as hot or hotter than our Sun. For nuclear fusion to occur in the Sun's core *(the only place in the sun where it happens),* hydrogen nuclei must fuse, kicking off a chain reaction to form helium nuclei, leading to the release of enormous amounts of energy in the process.

However, for hydrogen to fuse and ultimately produce *helium-4,* first, a crucial step in the chain needs to occur. The fusion of protons between two hydrogen nuclei yielding deuterons. It turns out that the force of repulsion between positively charged protons is so strong that even a single proton-proton pair cannot be fused, let alone kick-off nuclear fusion. And without nuclear fusion, the Sun wouldn't be a source of energy, making life on planet earth impossible. Yet, it seems our stars have aligned, hydrogen somehow converts into helium, and the Sun blazes in all its glory. . .
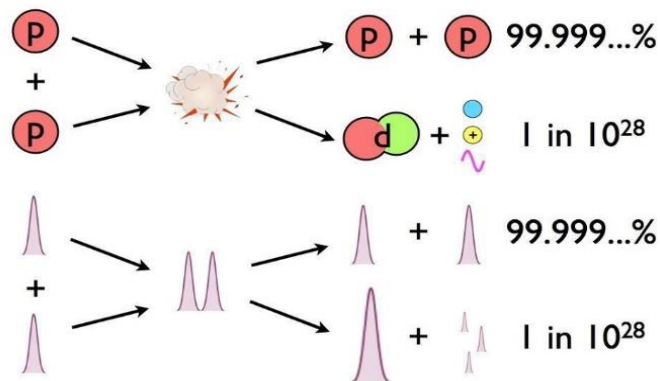

Figure 12: Quantum Tunneling enabled stable Deuteron production in the Sun's core. (E. Siegel/Beyond The Galaxy)

The Secret – Q*uantum Tunneling*. Each proton is a quantum probability wave function remember. When two proton *wave functions* meet, they overlap, ever so slightly letting parts of their wave nature tunnel through, even when the classical laws of electromagnetism *"like forces repel, unlike attract"* would otherwise keep them apart. This quantum-ness lets a tiny fraction of protons to tunnel through the wall of repulsion to form *"deuteron,"* thereby initiating the chain of nuclear fusion in the Sun's core. The probability of quantum tunneling occurring in the Sun's core is determined to be in the order of 1-in-10^{28}, the same odds of you winning the lottery three times in a row.[12]

Think about it, "Probability makes Sunshine and Quantum Tunneling makes Probability, and God does play dice…, quite vicariously I must add." [T]

. . .

Δ. "Shut Up and Calculate" - Bits vs. Qubits

> *"Everybody who has learned quantum mechanics agrees on how to use it. "Shut up and calculate! "*
>
> - *N. David Mermin, 2018* (Making better sense of quantum mechanics I, p 2)

The **"Mill"** was a calculating unit, built in the 1870s', analogous to a Central Processing Unit (CPU) that powers a smartphone or even a supercomputer today. This machine was something new for its time, and its most revolutionary feature was the ability to program operations via instructions on punched cards. Fast-forward 150 years, and we are yet again at the cusp of something new and revolutionary. Depicted to the right of Babbage's mechanical computer is an IBM Q 50-qubit quantum computer. One thing is sure; it seems computational infancy has a common denominator – its bulky aesthetics…
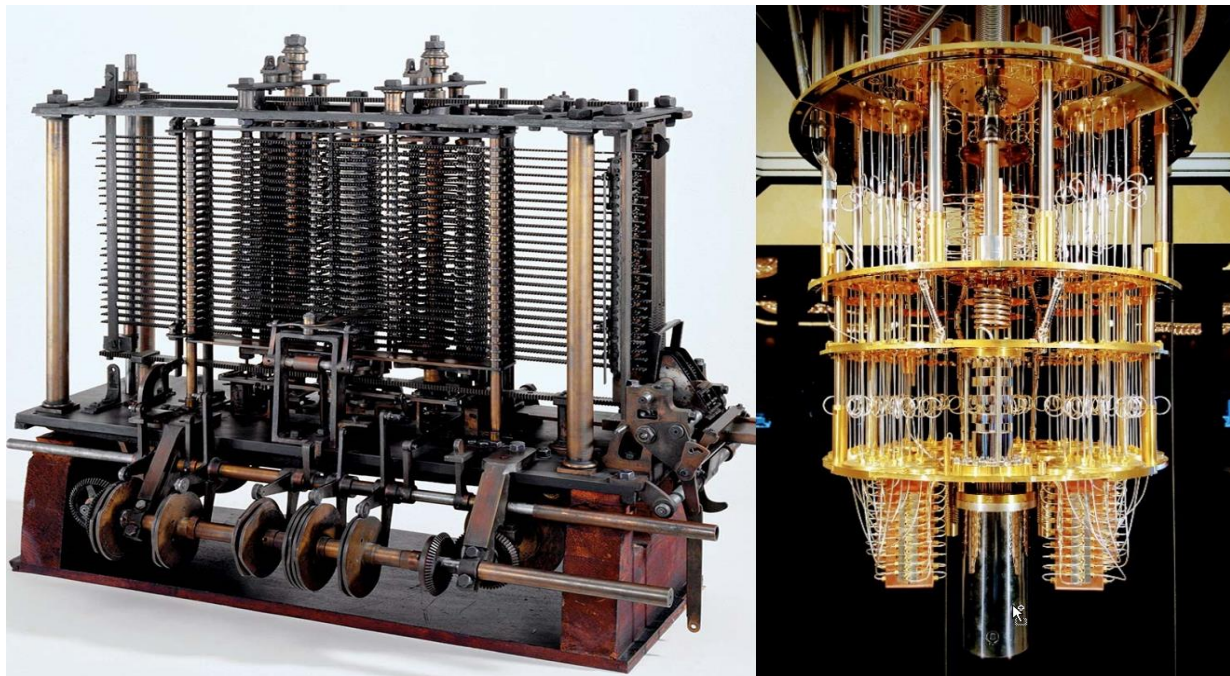
Figure 13: (Left) A portion of the "Mill" at the time of Babbage's death in 1871. (Science Museum London brittanica.com); (Right) IBM's Q quantum computer. *(Photo by Lars Ploughman, CC-BY-SA 2.0)*

The field of quantum computing is still in its infancy. But, over the years, the technological impediments to building a general-purpose quantum computer has been yielding rapidly to scientific rigor and ingenuity, buoying our quantum spirits. Quantum computing exploits the deeply embedded physics of nature, much of which is yet to be *demystified*. But rather than contemplating the metaphysical aspects of quantum mechanics here, we will focus on following Mr. Mermin's[T] famous axiom on quantum mechanics – "*Shut up and Calculate!* "

Straight to the "Fun" -da- "mental"-s

QuBits are the fundamental elements of information in quantum computing. A bit in the "classical" sense is either a 0 or 1. Measurement of the state of a bit is straightforward; it's in one of its possible two states. But for qubits, the measurement is a crucial aspect of its mathematical description.

To understand Qubits, we once again call upon an old sub-atomic friend – **the electron**, precisely its *"SPIN."* Steering clear of venturing into "Spintronics []," for simplicity, we will define *"Spin"* as a quantum mechanical property of elementary particles, often perilously equated to the spin of earth around the sun. This property gives the electron a bar-magnet like equivalence when placed in the presence of a magnetic field, as depicted in *figure 14*.
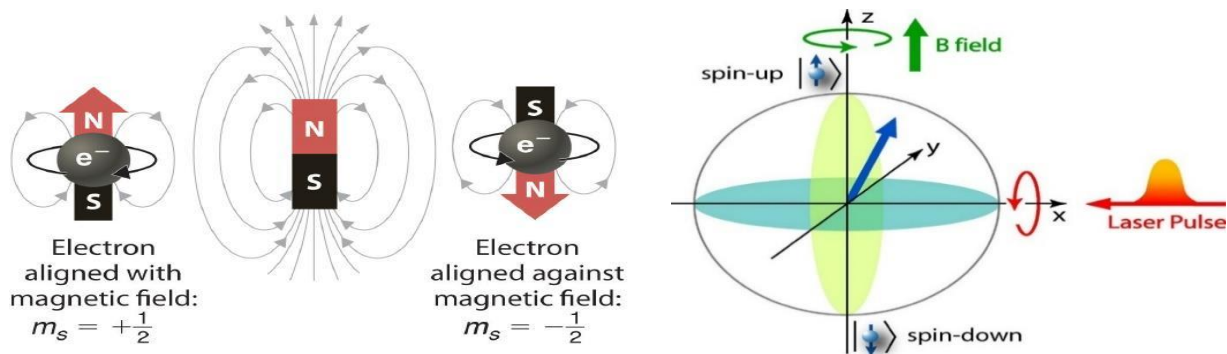

Figure 14: (Left) An electron spin aligned with a magnetic field. (Right) An electron in superposition represented by the bold blue arrow in the Bloch Sphere. Spin-up & spin-down states are at the North and South poles.

Spin qubits use the property of the "spin momentum" of a particle – like an electron to store information. When a particle – i.e. an electron or neutron – is located in a magnetic field, its spin can take either a spin-down *(low energy)* or spin-up *(high energy)* state. A semiconductor nanostructure with a single electron particle may act as one qubit, with the electron's spin-down and spin-up states representing "0" and "1," respectively. In quantum mechanics, by convention, we use the Dirac-*ket* notation style to denote the *Spin Up* as **"|1⟩** or |↑⟩**"** and *Spin Down* as **"|0⟩ or** |↓⟩**"** pronounced *"ket 1"* and *"ket 0"*, respectively.

For an electron to spin from " |↓⟩ to |↑⟩ ", it needs the energy to make the transition from its low energy state. One method of effecting this transition is by pumping a *pulse of laser* whose resonant frequency matches that of the electron in a magnetic field, thus exciting the electron to a higher energy state. Using this principle, we could use a variation of the laser pulse to spin the electron into a state between " |↓⟩ and |↑⟩," a unique quantum superposition of the spin up and spin down states with a specific phase. This state is depicted by the bold blue arrow in *figure 14*. Now we have a qubit that can take the two classical states of 0 or 1 and any superposition in between the two states. A quantum computer exploits this property to its advantage.

At this point, it becomes indispensable to make use of a mathematical model that can describe the quantum spin using probabilities and vectors. A single qubit can be defined by a two-dimensional vector and can hold all the information needed to describe the one-qubit quantum state. The orthonormal basis corresponding to the spin in the vertical direction given by *z-axis,* calling it the computational basis, is denoted as, $|\downarrow\rangle$, $|\uparrow\rangle$ where;

$$|\downarrow\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } |\uparrow\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Other valid vector state bases could be very well used for qubits. Some equally valid examples for representing the qubits are, but not limited to, the following basis vectors,

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \Big| \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \quad \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}; \quad \Big| \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|\psi\rangle = cos\frac{\theta}{2}|0\rangle + e^{i\varphi}sin\frac{\theta}{2}|1\rangle$$
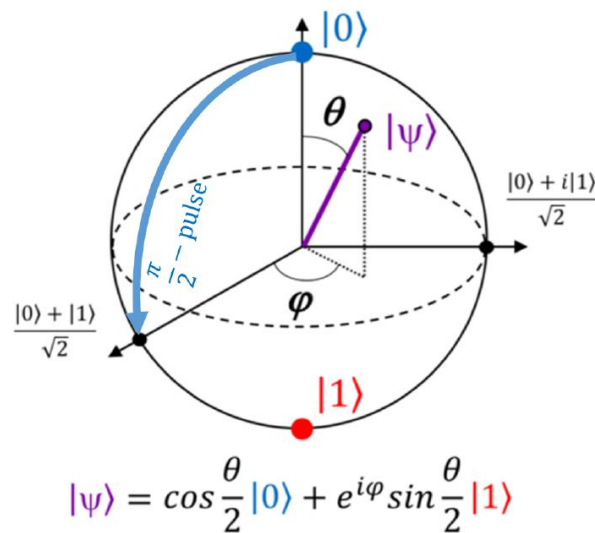
Figure 15: The Bloch sphere provides a useful means of visualizing the state of a single qubit and operations on it.[14]

The *Bloch Sphere* is a useful mathematical construct for thinking about measurements made to a qubit. It not only lets us record the amplitude of the measured probabilities but also gives us a view into the phase transitions caused by the vector state. While the classical bits can only take the "North Pole" or "South Pole" positions, the rest of the sphere's surface remains inaccessible to the classical bit. Unlike the bit, the qubit can access every point on the surface of the Bloch Sphere. Each point can be an arbitrary position for the qubit vector to point to in a superposition. So, if the vector points to specific coordinates on the surface of the sphere in the upper hemisphere, the probability of detecting a "**1**" is higher on measurement, and if the vector arrow points to the southern hemisphere, the likelihood of a *'0'* o is higher.

Recall that the measurement will collapse the wave-function. When we measure, the qubit will always find it in either the spin-up or spin-down state, i.e., 1 or 0. Before we measure, however, the particle will be in a superposition spin state given by the linear combination of the *kets* |0⟩ and |1⟩ denoted by the equation below

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

where **α** (alpha) denotes the probability amplitude for getting a zero on measurement and **β**(beta) is the probability amplitude for getting a one on measurement. Note these are probability amplitudes and not the probability. The probability is calculated by taking the absolute value of **|α²|** and **|β²|,** and those probabilities must normalize, meaning the total probability must add up to one.

$$|\alpha^2| + |\beta^2| = 1$$

The equation below denotes the geometrical representation of the qubit in terms of the Bloch Sphere.

$$|\Psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

The solution to **φ** defines a point on the Bloch sphere, while **θ** provides the phase of the vector for single qubits. The Bloch Sphere is an excellent mathematical tool to help determine the qubit's state transformations. Representing the qubit state as a point on a sphere also helps with visualizing the concept of quantum gate operations. Superposition is a probability distribution. The "amount" of superposition, *i.e.* how much "zero" and "one," mathematically called amplitudes, can be specified as a positive, negative, or even complex numbers. The fact that amplitudes can be negative opens the possibility of interference: positive and negative amplitudes can combine to suppress a probability, very much like the crests and troughs of water waves can cancel each other out. The power of interference in quantum computation is an essential factor that differentiates it from the classical computation.

Paradoxically, there can be an infinite number of points on the Bloch sphere surface, implying that the qubit can represent *infinite* states of superposition, enabling us to store unlimited amounts of data. However, it turns out; this is not the case because the very act of measuring the qubit collapses the superposition, always yielding either a 0 or 1. And post-measurement, the state of the qubit, is changed to the state consistent with the measurement result. For instance, if we measured the qubit and got a '1', then even after we are done measuring the qubit, the state remains as 1. This behavior is still a quantum mystery. That begs another question; what about the information in a qubit if we *do not* measure it? Well, how can one quantify information if it cannot be measured, right? But nature it seems, does this quite efficiently, somehow keeping track of a great deal of *'hidden information'* as it evolves a closed quantum system. Understanding this hidden quantum information is what lies at the heart of quantum mechanics, making it a powerful tool for information processing[7].

From Classical Gates to Quantum Registers

Logic gates are the fundamental unit building blocks of digital electronics. In addition to performing logical operations, gates can be used to perform arithmetic and build memory, otherwise called digital "flip-flops" to store data. Typically, by arranging the gates with a feedback mechanism and a 'clock' signal, basic flip-flop types are created, which in turn can be used as building blocks for creating multi-bit registers. The set of operations run on gates and registers to manipulate logic states is what makes up the logical flow states called algorithms. This scheme can be extended to both classical and quantum computations.

An important concept here is that of reversibility. Reversibility in computing implies that no information about the computational states can ever be lost, so we can recover any earlier stage by computing backward or un-computing the results[15]. This property is known as logical reversibility. Most gates are not reversible, but some special gates can be designed to be reversible. An example of reversibility is demonstrated by a "*Controlled NOT*" gate or **CNOT** gate.

If **x, y** are inputs then $f(x, y) = (x, x \oplus y)$. **x** acts as the control bit. If you only had the output (x y) and x, you could reverse the process and get the inputs. Something that only reversible gates can get you. The CNOT gate isn't just invertible, but it also is inverse.

So, for example, if you connected two CNOT gates in series, where the output of the first gate becomes the input of the second gate, the output from the second gate will be identical to the input you fed into the first gate. The truth table for a CNOT gate and a quantum CNOT gate is equivalent, but superposition will allow for more fun outcomes.

CNOT			
Input		Output	
x	y	x	$x \oplus y$
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

A circuit for *CNOT*.

Figure 16: Reversible CNOT gate.

A one qubit system is represented as a single wire. Its initial state is 0 and is conventionally using the standard bra-ket notation**|0>.** The elements that live on the wire are single-qubit gates, which are simple rotations around the Bloch sphere that transforms the state of the vector. A measurement is a non-reversible process and will output one bit of information for each qubit.

Figure 17: A single qubit wire in the circuit model picture.

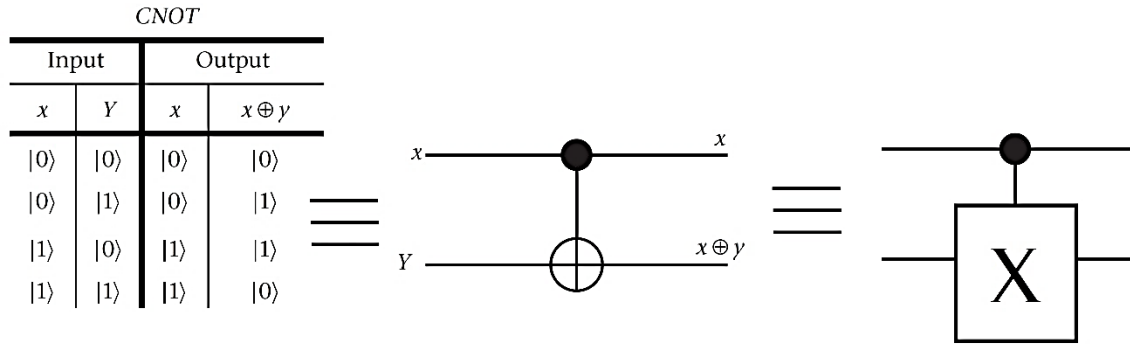**Figure 18: Quantum Gate - Controlled NOT (U$_{CNOT}$) circuit diagram.**

A circuit representation of a quantum system **U$_{CNOT}$** is shown in *figure 18*. If "x" the control qubit is set to 0, then the "y" target qubit is left unchanged after transformation, but if x=1, then the "y" is flipped. Astute readers will note that the truth tables for the classical CNOT gates are equivalent to the quantum CNOT. While that may well be the case, quantum superposition allows for more amusing outcomes. If the control qubit is placed at the equator of the Bloch sphere, i.e. there is an equal probability of |0> and |1>, and the target qubit is in a simple |0>, the following happens *(see the outcome in figure19)*
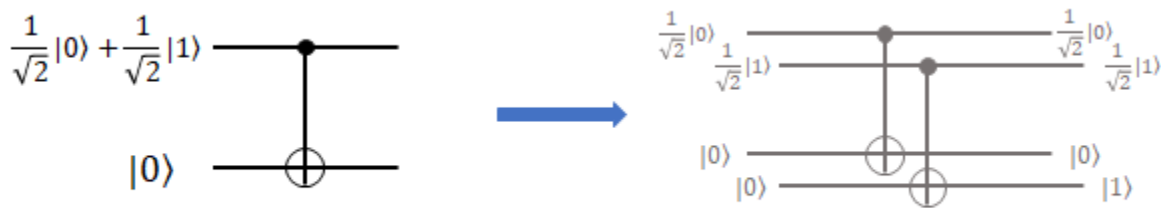
Figure 19: A U$_{CNOT}$ quantum gate in superposition. (Image by Quantum World Association)

Having the control qubit in a superposition, causes the target qubit to flip only in one of the branches of the computation, but not on the other. The resulting state is commonly known as an entangled Bell state. The **U$_{CNOT}$** gate is a universal quantum gate given the fact that we can reduce any operation on a quantum computer to some combination of **U$_{CNOT}$** gates, making it the quantum parallel of a universal **NAND** gate.

Quantum gates make quantum circuits that become the building blocks for a quantum computer. And all unitary quantum gates are always reversible, an inherent property of unitary matrices from mathematics. Thus, a quantum gate can always invert the action of another quantum gate. This property of reversibility will be an essential facet that will enable quantum algorithms to harness the power of quantum computations.

ξ. Architectures & Algorithms – A Quantum Zoo

"Quantum Computation will be the first technology that allows useful tasks to be performed in collaboration between parallel universes."

-    *David Deutsch, 1997*(The Fabric of Reality;C.9.p.136)

A candle and an electric bulb both perform the same base function – emitting photons. Nevertheless, it would be a trivial act in naivé to compare the fundamental mechanisms of the two agents of light. No analogy can justify the principle behind the blackbody radiation of the bulb to the candle wax. To understand the bulb, you have to forget entirely about the candle.

Understanding quantum computing requires a fair bit of re-wiring of our neurons and a fresh temperament for a different type of computing. This effort is much more than just building a faster or smaller computer. It involves channeling a whole genre of complex computation beyond the limits set by the physics of classical machines. If a computer were tasked with solving a maze, it goes about finding the best route methodically, applying step-by-step brute-force routines trying each path one by one. No matter how amazing *Deep Blue* was – *the supercomputer that ousted Kasparov from the grandmaster throne;* it was a product of brute force application of computing resources. A quantum computer instead will calculate every path out of the maze parallelly using superposition and entanglement to arrive at the solution by carefully balancing a "*dance of waves of probability amplitudes and their square roots…*"
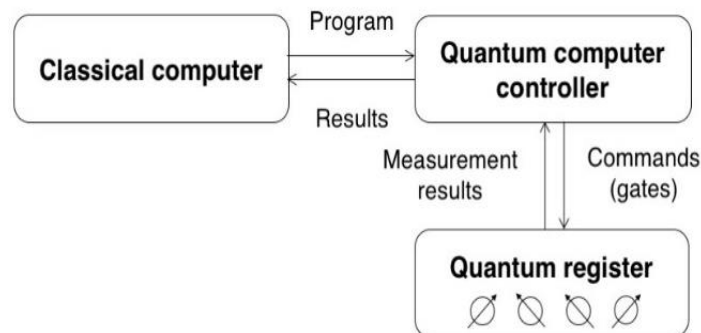
Figure 20: A basic model of a quantum computers' building blocks

Far from the dreary view of the total annihilation of classical computers, the current consensus on how to architect a quantum computer seems to agree on the fact that at least for now, a quantum computer will have to make-do in a mutualistic symbiosis with its classical counterparts. There are two critical reasons for this, one has to do with the fact that to interact with the quantum mechanical system, we humans need an interface that can efficiently deliver the results of our "act of measurement," and the other reason has to do with the time to *decoherence.*

Building Blocks of a hybrid architecture

A proposed view of the quantum processor unit *(QPU)* pitches it as a new accelerator unit in a heterogeneous multicore architecture. This is typical of supercomputing models and is based on the idea that an application can be designed to leverage multiple computational kernels with each capable of executing specific accelerator modules. There can also be various competing qubit technologies like qubit dots, quantum annealing, semiconducting and superconducting qubits, NV-centers, all co-existing within the same enclosures as depicted in *figure 21*.
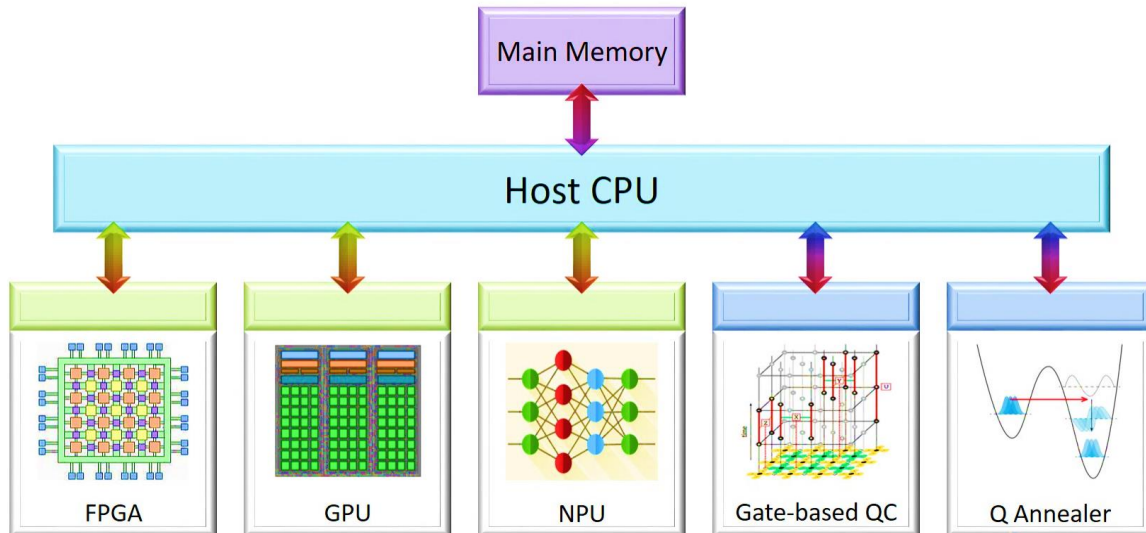

Figure 21: System architecture with heterogeneous accelerators.[16]

Like conventional systems, quantum computers need memory to store qubit states. These memory registers hold the quantum states on which operations are executed. But there is a fundamental difference between memory systems in conventional computers as opposed to quantum memory. The quantum memory is relatively passive compared to Random-Access Memory (RAM) in a regular computer. Quantum data within the qubits typically last only until the quantum operation acting on it.

The role of the classical CPU in the hybrid quantum architecture is noteworthy. Quantum algorithms are held in memory of the CPU, which instructs the gate devices controlling each qubit. There is a carefully orchestrated effort between quantum and classical components that are crucial for making final measurements and deriving meaningful output from the quantum accelerators. Input-output operation is a concerted effort between the CPU and QPU pairs, which directly impacts the performance of the hybrid architecture. The efficiency with which quantum data can be moved around inside the quantum computer, combined with the structure of the quantum algorithm, determines how quickly we can solve an individual problem.

Breaking down the system stack

The topmost layer consists of the Quantum Algorithms (QA), for example, an algorithm for simulating the molecular structure changes in a metal during heating or mathematical factorization for prime

numbers, etc. This layer holds the best promises and where the most significant opportunity lies for organizations worldwide. The next layer constitutes the application programming layer which would be most suitably run on the classical processors in the microarchitecture design. Q# by Microsoft uses its Quantum Development Kit, QCL (Quantum Computer Language), whose syntax and data types resemble those used in C programming language. QCL is the most advanced implemented quantum programming language. IBM has developed the Quantum Information Software Kit (QISKit), which is a full-stack library to write, simulate, and run the quantum program.
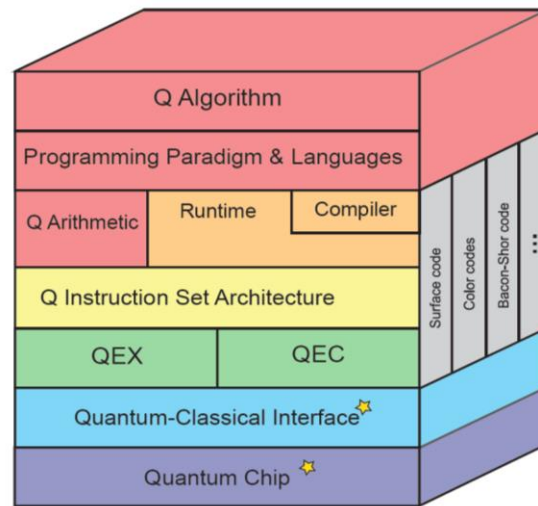

Figure 21: Layers of a Quantum Computing System Stack Model.[17]

The quantum arithmetic layer, as the name suggests, does the arithmetic while the runtime and compiler act as translators taking inputs from the top layers and compiling them into the instruction sets. The instruction set describes what operations the quantum device can execute and is crucial in leveraging the parallelism that quantum mechanics offers. The bottom layers involve the specifications and definitions of the physical medium, the qubits, and the registers and gates built from them. This function will be heavily dependent on the QEC/QEX layers that will provide the error correction and optimization attributes. A quantum device can achieve massive parallelism, in principle, due to the probabilistic and non-deterministic computing style of a quantum accelerator. The computation is not unidirectional and will involve multiple algorithmic runs to optimize the probability curves to generate the most probabilistically accurate results. This approach is very different from the sense of classical computation. As the physical quantum devices evolve and mature toward hyper-scale fault-tolerant quantum systems, so will the quantum programming languages and algorithms evolve, driving up the efficiency and formalizing the quantum stack.

Quantum Algorithms

The value of quantum algorithms is in the utilization of the non-deterministic characteristics exhibited by quantum systems. It's important to note that they are not merely regular algorithms that have somehow been sped up; instead, they involve quantum ideas to see the problem in a new light; the quantum algorithms use ingenious methods of exploiting underlying patterns that can be seen from only the quantum viewpoint.

Quantum computers enable a new class of algorithmic complexity with different characteristics than their classical equivalents. This enables the possibility of processing exponentially large quantities of information in polynomial time. What's polynomial-time you ask? Without getting into the weeds of time complexity, all algorithms take a certain amount of time to run, almost always growing with the computational complexity. When this time grows *linearly, quadratically, cubically, etc*. it's said to be in polynomial time; on the other hand, if the time to solve a problem grows *exponentially,* its exponential time. Problems are classified into complexity classes. A simplified view of the complexity classes is given in *figure 22.*
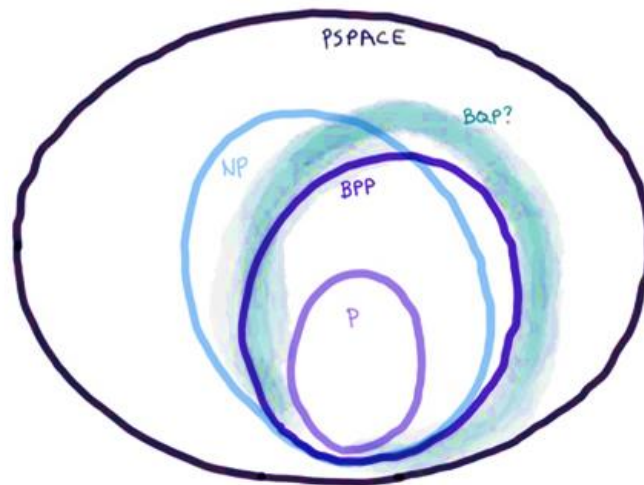

Figure 22: Complexity Classes. (Image by infoq.com)

P contains the class of problems that can be effectively computed by an algorithm deterministically in polynomial-time. *NP* is the complexity class that deals with problems for which, regardless of the complexity, we can verify a given solution as a real solution in polynomial time. ***Bounded-Error Quantum Polynomial-Time (BQP)*** is the class of problems known to be efficiently solvable by a quantum computer. It has a classical counterpart called *BPP*. The relationships between *P, NP, BPP*, and *BQP* is a vividly engaged field of computer science meets complexity theory that is a subject in itself.

It's noteworthy that of late, there has been some misplaced pragmatism around quantum algorithms. Commonly popular descriptions for quantum algorithms often exalt quantum algorithms as being faster, better, and stronger than regular algorithms merely because they use quantum phenomena like superposition to their advantage. Clearly, this is not the complete picture, and there is more to quantum algorithms than just putting everything into a superposition of states. The real art of constructing these algorithms lies in being able to manipulate these superpositions so that, when we make measurements, we get a useful answer. We will briefly look at two quantum algorithms to get an understanding of what they constitute and how they can be beneficial to the field of quantum computations.

Quantum Factors

What are the prime factors of 73? Take a minute to calculate the answer to this question for the quickest means to garnering some appreciation for the problems of factorization in the grand scheme of computational complexity. Factorization is simply finding two numbers whose multiplicative product gives precisely the value we are trying to factor, i.e. #73 in our case. As the number of digits piles on to the right, you will find that arriving at answers become asymmetrically hard. Factoring is an example of a one-way function wherein if the divisors are known, finding their product is easy, but the *vice versa* of finding the divisors from the product becomes a challenge in polynomial time. The highest RSA number factored on a classical computer was RSA-768, which had 768 bits, and took two years to compute.[19] To put the number of digits into perspective, we know the value of pi with the accuracy of almost 2700 **"billion"** decimal places. It took about *131 days* to complete the calculation and one terabyte of storage. Trying to download it would take ten days, and reciting it aloud would take about 49,000 years. And yet, factorization has us struggling with 768 bits of RSA.
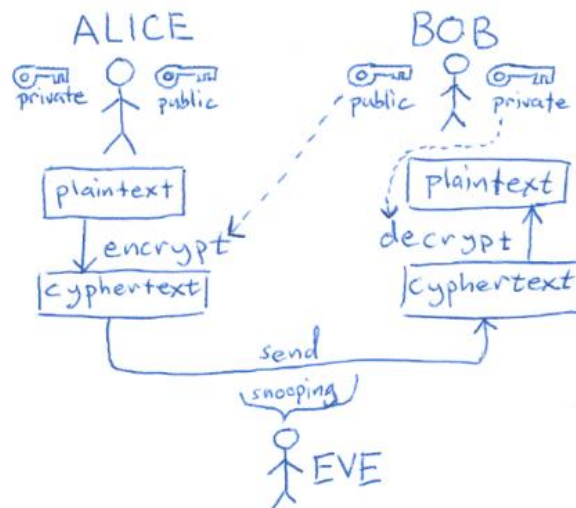
Figure 23: Asymmetric (Public-key) Encryption. (Image by USNA [20])

This asymmetric difficulty has made factoring a core part of most public-key cryptography. The prime products of two numbers can be used to generate a public key, which senders use to encrypt messages and recipients then decrypt on the other end via a private key that's created based on one of the factors.

Now, *"breaking an RSA"* – easier said than done – reduces to finding the prime factors of a large integer, and Peter Shor in the mid-1990s developed a quantum algorithm that opened up a possibility of breaking RSA cryptography. Shor's cleverness was to use an iteration of classical computing to determine the period of a function synonymous with the frequency of occurrence within a given sequence, which then mathematically starts to look like a wave function with a frequency. "Fourier Transforms" is used to turn a function (like a wave signal) into its constituent frequencies. The quantum Fourier transform picks out frequencies that make up a time series using quantum superposition to effectively measure the function at multiple units of time, and then interferes the waves so that the right solutions amplify, while the wrong solutions cancel out paving the way for a measurement to yield

to the correct answer with a high probability. Understanding the essence of Shor's algorithm without math is no easy task, and this is the extent to which we will pursue this matter here.

Quantum Search

Grover's search algorithm was the second hit in the quantum algorithm album two years later after Shor's number one single. Grover's algorithm is like a *"database search with a lemon twist."* Take out the phonebook if you still have one lying around somewhere, else imagination works fine too, and try to find a name in the phone book considering that you only have the person's phone number. If you had a classical computer handle this task on an average, it is estimated that it would have to search through at least half of the data to locate the name. *No shortcuts - period*. Unless you had a quantum computer, and Grover was your friend. Grover's Algorithm would allow you to search an unsorted database in $O(\sqrt{N})$ time as opposed to the classical $O(N)$ time. While this is not the same exponential speedup as Shor, it still is a quadratic speedup and has properties that make it lucrative for a broad spectrum of applications. Although Grover's algorithm identifies as a "searching " function, it can very well be described more accurately as "inverting a function".[21] Let's say we evaluate the function *F'* = f(x) using Grover's search algorithm; then, this allows us to calculate x when given F.' So, in essence, we could have sub-functions that could produce a specific value for T if x matches an entry in the search database and another subfunction of F' for another x in the search database. Quantum algorithms are inherently probabilistic, and Grover's algorithm follows suit. The right answers are derived after multiple algorithmic runs are performed. After every execution, the answers add to the probability. With each iteration, the solutions with the least possibility need to destructively interfere and cancel each other out for quantum algorithms to give any interesting speedups.

There is yet to be found a more generic underlying principle that can be used in quantum algorithms for what is known as the universal quantum speedup. This is a hard problem to solve, but the lack of such a general algorithm must not halt us from realizing the value of existing paradigms and the extended applications it promises for the future.

λ. The Promise of the Future

Miracles or Mirages

"The real question is not whether machines think but whether men do. The mystery which surrounds a thinking machine already surrounds a thinking man."

— *B.F. Skinner, 1969.* (Contingencies of Reinforcement)

The destiny of *quantum computing* seems to be, quite ironically, "e*ntangled in a superposition"* [T] itself. The views on its realized potential can fluctuate between a *universally game-changing* paradigm to an exponentially decaying *impossibility* – depending on which camp of the combinatorial flavor of believers and skeptics one is referring to. In principle, a *300-qubit* quantum computer could perform more calculations at once than there are atoms in the observable universe.[22] This theory can be an argument for the power of quantum computers or one against it, showcasing its extravagance of imagination waiting to buckle under the '*force majeure'* of reality. Personally, when the *physics* gets tough, one finds solace in *philosophy*, until, the vagary of *philosophy* forces one back into *physics*. In this context, of particular mention among my favorite wordsmiths is *Daniel Dennet*, whose philosophy on *"failing successfully"* is an anthem of the unique value of errors. What kind of a world would we be living in if we only ran the races we were sure to win. It is better to have begun and failed than to have never begun at all.

It would be entirely fair to say that over the last two decades, there has been significant progress in quantum information theory and quantum hardware architectures. This positive momentum has opened up new avenues of *entrepreneurial pursuit*, evident in the uptick among startups and industry leaders alike focused on quantum computing. It may be entirely the case that quantum computing is impossible or impractical due to some fundamental reason, but it is still far too early to quantify this possibility and unfair to announce that we'll never realize the promise of quantum computing. There is no reason why we should not continue even if there exists *a quantum of probability* to fail successfully.

The **upside** of succeeding, on the other hand, is simply phenomenal, and stating that quantum computing will change humanity forever will be a gross understatement. The potential applications that can be revolutionized by quantum computers are numerous. While it is common to find many writings online with a "Q" for quantum slapped in front of every possible combination of words in the "*Queens English,"* I am going to make a conscientious effort to try and focus on the possible miracles and try to steer away from the myriad myths and mirages. I believe at this time; there are three primary areas – Life Sciences, Technology and Communications where a quantum advantage can deliver maximum impact provided; we continue to improve on the quality of quantum computational accuracy.

So, what does one mean by the quality of computational accuracy? All computation systems have to contend with the efficiency of results. We live in an analog world where interference between systems is a natural state of being. With classical computers, error correction has been very successful, but with quantum computers, this gets a bit tricky to solve, given that we are operating at the subatomic scale.

Whether the qubit substrate is photons or electrons or ions, we must contend with *Noise. Noise or Error* is environmental interference such as temperature fluctuations, mechanical vibrations, or stray electromagnetic fields that can weaken the correlation between qubits, degrading the reliability of the quantum state machines. This error could limit the size of quantum systems and compromise the spectrum of computations that they can perform. Noise is usually compensated for with error-correcting routines, but such methods require dedicated qubits that could add errors of their own, setting limits of how many error-correcting qubits per compute qubit could be packed as a unit. Thereby adding overhead and putting physical limits on the size of the quantum systems. But the spirit of human endeavor is infectious, and the noise hasn't slowed us down. On the contrary, while efforts to cancel the noise are underway, we parallelly are working on systems where the noise/errors could be an added feature, enter Noisy Intermediate-Scale Quantum (NISQ) computation.

Quantum Chemistry – Bugs are a Welcome

As the number of atomic bonds in a molecule increases, the complexity of simulating these systems becomes so mathematically complex that conventional computers are not a practical computational choice. The property that makes these molecular simulation problems unique is the fact that unlike fields like cryptography or communications where errors can cause systems to lose coherency, noise can act as a physical feature. NISQ computers are a perfect fit for chemistry, providing a possibility to simulate these extremely complex molecular interactions without having to wait for hyper-precise quantum devices. Think about it, "Noise," is simply the interaction of physical and natural systems with its environment. In chemical reactions, the interaction with its environment manifest as thermal fluctuations. These fluctuations, in computational terms, is referred to as noise or error. So, a quantum device that comes with inbuilt noise would be a more accurate representation of the physical environment in which chemical systems exist. Chemistry with quantum qubits is inherently more representative of a naturally occurring system, making NISQ-devices a perfect fit for these purposes.

There is tremendous momentum in the material design and discovery area where, until now, there was little to no computer-based optimization because it simply was not a meaningful exercise with classical computers. Natural materials like crystal and semiconductor materials all follow the laws of quantum physics, making them perfect for quantum computer-aided design and discovery. In 2018, Cambridge Quantum Computing ("CQC") announced in collaboration with JSR Corporation ("JSR") that they had successfully implemented state-of-the-art quantum algorithms to calculate the excited states of molecules that model multi-reference characteristics.[23] This development is a prime example of the quantum advantage, since tackling multi-reference *states* has traditionally failed both quantitatively and qualitatively in the past. The hardware used for this algorithm was a *20-qubit* IBM Q quantum computer.

Finally, there are multiple applications in catalyst identification, molecular biology, and drug discovery that are prime targets to benefit from the quantum advantage. While the next few years could be defining moments in the quantum evolution of applications, there still are challenges and milestones to achieve before the definitive *quantum takeover*.

Quantum Enhanced Machine Learning

Conceptually, *Machine learning* (ML) *and Deep Learning* (DL) is primarily about finding and recognizing patterns, thereby deriving actionable information from data. A wide variety of ML and DL algorithms work on mathematical models, like matrix operations on vectors in a 2^N dimensional vector space. Incidentally, quantum mechanics lives and breathes (figuratively) in the vector space. Furthermore, we know that quantum qubit operations in 2^N complex vector space perform matrix transformations exponentially faster than classical computers.

The outlook on using quantum computers for machine learning has come into the spotlight lately because we have a handful of quantum accelerators that look very promising, thus enabling their adoption for AI workload acceleration. Many quantum processor architectures resemble the hardware used for special-purpose Application-Specific Integrated Circuits (ASICs) that implement a subset of quantum algorithms. This approach can have us running machine learning and deep learning algorithms specialized for quantum devices that target specific problem sets in the near-term. Optimization problems, for example, are already prominent in quantum physics, wherein finding the lowest energy point in an energy landscape is typically the goal. This it turns out is the fundamental paradigm in adiabatic quantum computing and quantum annealers.
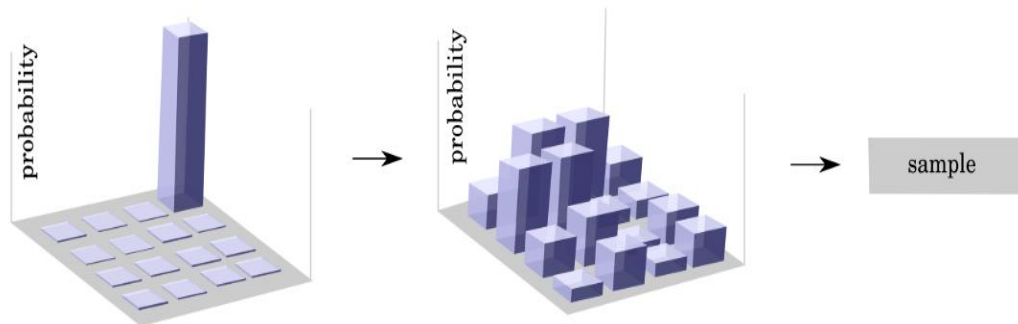

Figure 23: Probability Sampling

Sampling is another field in which machine learning and quantum processing go hand in hand. All quantum computers act as samplers, creating simple probability distribution samples via measurements. Quantum devices are, therefore, up-and-coming assistants for sampling-based machine training.

ML techniques like *Clustering* and *Recommendation-engines* are prime targets for quantum enhancement. For example, Lloyd's algorithm to solve the k-means clustering problems QML can provide a means of significant advantage to its classical counterparts. *Quantum Neural Networks* is a technique employed in deep supervised learning to train the machine to classify data, recognize patterns and images. The principle used leverages qubits and rotation gates to operate the network analogous to the neurons and weights as used in a classical neural network to obtain the training parameter that provides a minimum error. The progress in *quantum deep learning* is very promising.

Besides enabling the existing AI and machine learning models and techniques, quantum machine learning can lead to entirely new models for training machines, both classical and quantum alike. Machine learning could eventually become a standard component for building quantum computing

hardware. Lately, there is more and more evidence that quantum machine learning is evolving from being a subtask of quantum computing to become an approach to quantum computing itself.

Nevertheless, there is yet not a general theory to analyze and engineer new quantum machine learning algorithms. Challenges in the quantity of input data that the proposed implementations can handle and the quantum dynamics with memory, which simultaneously conserves its quantum properties, are yet to be addressed satisfactorily. Although quantum algorithms can provide dramatic speedups for processing data, they do not offer advantages in reading data. Sometimes the cost of reading data exceeds the value of quantum algorithms. These challenges need to be addressed for meaningful large-scale adoption of quantum techniques.
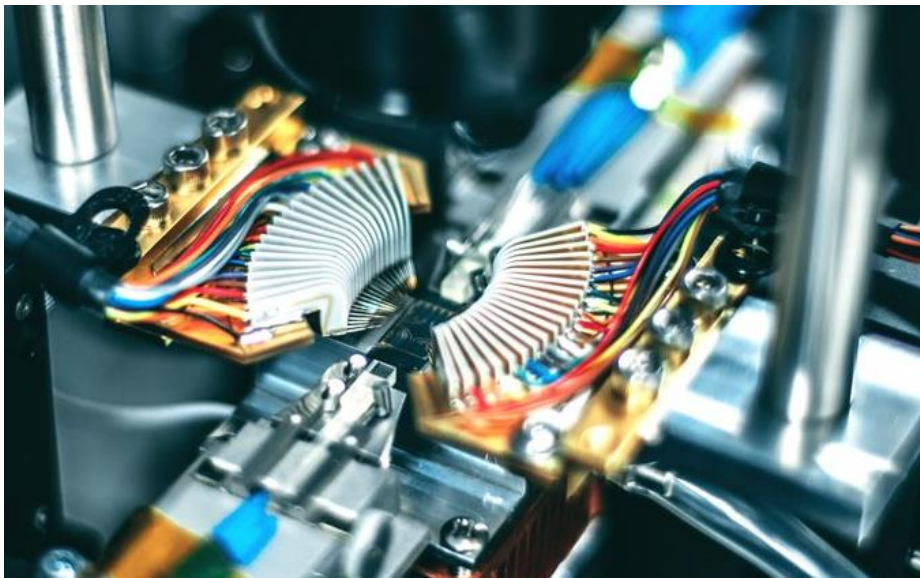

Figure 24: Quantum chip from Xanadu based on information in light beams (photons). Image by Xanadu Quantum Technologies Inc.

Quantum Teleportation and Quantum Internet

This topic may seem like it belongs under the *"Mirages"* category of quantum applications, but you will be amazed to see its real. Almost three years ago, a team of Chinese physicists launched a 1400-pound behemoth of a quantum satellite into space. This satellite has been humming all these years information encoded in entangled photons to and from quantum stations on earth. In 2019 physicists demonstrated quantum teleportation between two computer chips for the first time. Quantum Teleportation uses entanglement to transfer information instantaneously across vast distances. If you think this will soon be the answer to the long security lines at airports, you will be disappointed; at least for now (not that I am suggesting quantum teleportation will get us there). It only applies to a single photon and doesn't even transmit the actual particle, just information about its state through the quantum mechanical phenomenon of entanglement. The entanglement of qubits comes with some inherent qualities. The first feature is that it allows maximum coordination. Meaning any state change on one of the entangled pair is instantaneously reflected on the other. The second feature of entanglement is that it is inherently private. It turns out only two qubits can be entangled at a time. This makes the communication between an entangled pair inherently private and unbreakable.

Additionally, it is vital to consider the characteristics of quantum systems in general; a) quantum states cannot be transmitted over classical channels because measurement collapses the wavefunction, and b) the **'no-cloning' theorem** restricts us from copying quantum information. These characteristics boost the case for a quantum information system whose security is governed by the laws of physics instead of by mere computational complexity. Leading to the possibility of a network of quantum communication. *The Quantum Internet***.**
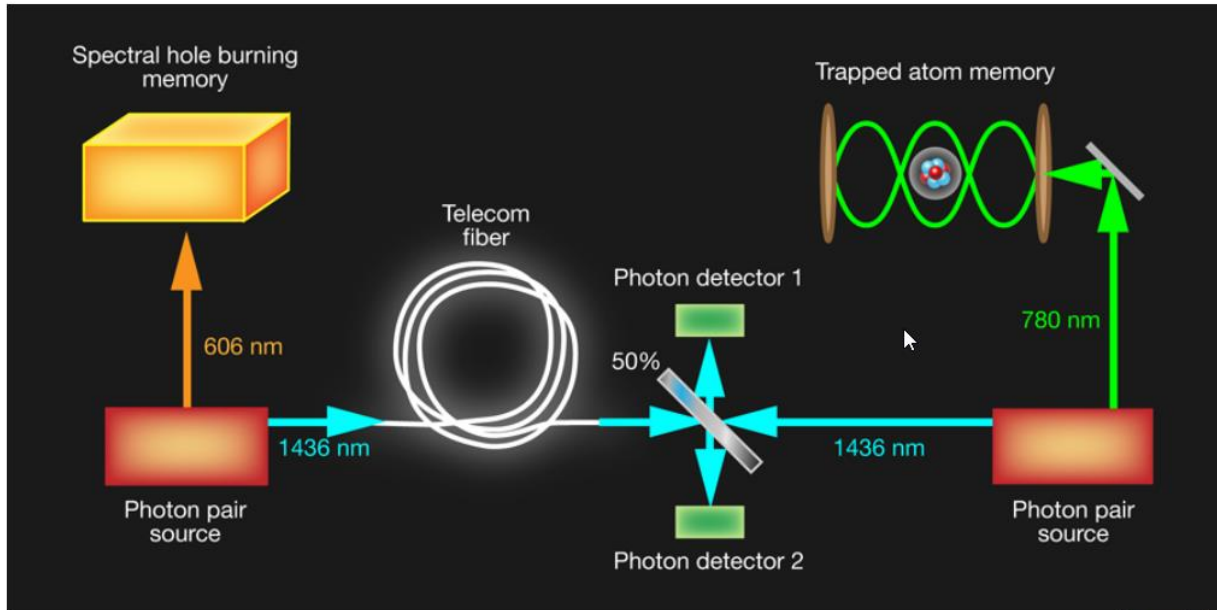
Figure 25: Illustrating the transmission of photons over internet fiber.[25]

Instead of 1's and 0's, qubits will be the transmission currency in the quantum internet. A high-level topology of a "Quantum internet [24]" is depicted in *figure 25.* First, we need a physical medium, **"a** *quantum channel*," for transmitting qubits. It turns out that we already do this today using fiber cables for our everyday *"vanilla"* communications. The next will be a means to transmit the qubits over extended distances. All physical mediums are lossy by nature, and so are quantum channels. Hence to reach longer distances, booster nodes called *quantum repeaters* are necessary. These repeaters are placed along the optical fiber connecting the quantum channel across switches allowing qubits to be transmitted over arbitrarily long distances. Lastly, there are the quantum processors connected to the quantum internet called the *end nodes*. Recall that the architecture for the hybrid model of quantum computing defined the quantum processor as an extension of a heterogeneous accelerator ecosystem managed by the classical *"CPU."*
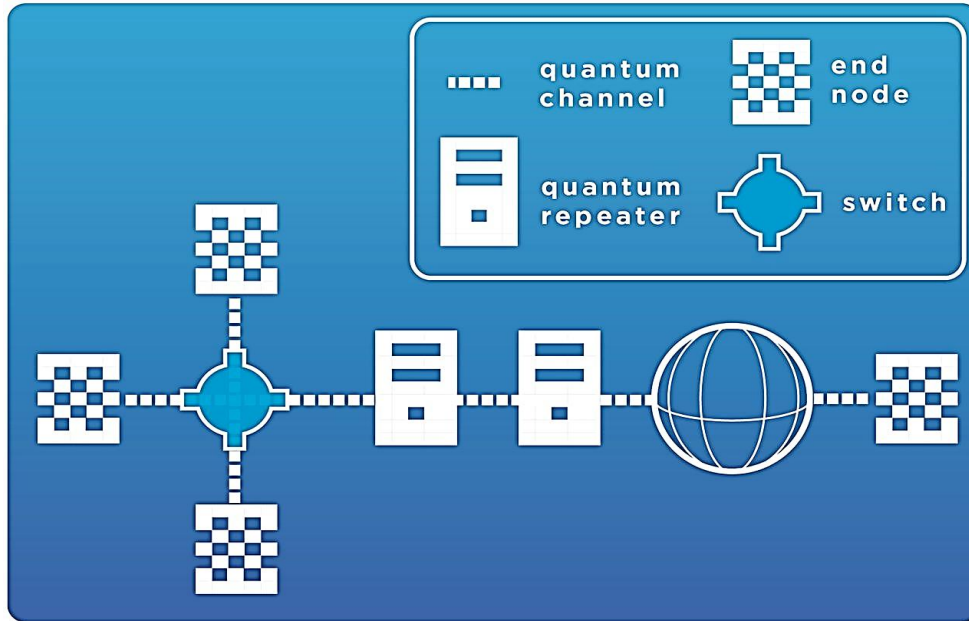
Figure 26: Three essential hardware building block components for a quantum internet [24]

The quantum internet is relatively in its infancy, and the applications that it would eventually be used for is anyone's guess, but one application evident is the ability to allow secure communications. Currently, the experimental status of long-distance quantum networks is at the repeater stage. Some repeater tests over long ranges have already been reported successful. The quantum internet is closer to fruition than we might expect waiting on the next quantum leap.

The Quantum gold rush…

We are thousands of qubits away from a general-purpose quantum computer, one that could deliver the quantum speedup we desire. Until recently, quantum computing had been a research effort in labs of universities. Lately, however, there has been an upsurge in funding being poured into quantum technologies. Investors and large enterprises alike are all banking on breakthroughs. But quantum machines are still likely to be decades away from achieving their true potential, and even then, realizing efficient algorithms to harness their abilities could be a challenge. If quantum progress stalls or slows down, it won't take much time for the buzz to turn into a bubble.

∞. The Theory of Everything from Nothing
A beautiful dream or simulated nightmare

"The first gulp from the glass of natural sciences will turn you into an atheist, but at the bottom of the glass, God is waiting for you."

— *Werner Heisenberg, (as cited in Hildebrand 1988, 10)*

"Emptiness certainly is anything **but** *Empty*." This phrase is an unusual choice to begin any discussion with but an extremely crucial realization that needs to be reconciled from the get-go. Just as you start to wrap your head around some of the finer aspects of the world of quantum weirdness, you are thrown another curveball. It is believed that the next revolutionary phase in physics could very well be about *Nothing* – the existential vacuum

Aristotle is famously attributed to the postulate **"horror vacui,"** translated as **"nature abhors a vacuum."** An atom is mostly made up of empty space — this is commonly taught in schools. So is the expanse of the universe — the vast infinite void. A famous analogy to the emptiness in an atom is to compare it with a large football field and equating the nucleus to a fly in the center and electrons orbiting the seating area. Unfortunately, none of this "empty space" prep talk is entirely accurate.

The vacuum is not really empty and is bubbling with energy and particles that burst in and out of existence. Emptiness is actually filled with countless "virtual particles." These virtual particles come into existence out of nothing in a void and are annihilated immediately, giving out energy. This, in and out of existence, extends throughout all emptiness, creating what is knows as a quantum field, giving rise to the *Quantum Field* Theory (QFT). QFT asserts that particles are simply manifestations of the virtual particle excitations arising due to the fluctuations in the quantum field that extends into infinite space. With its dependence on virtual particles and vacuum fluctuations, QFT has become one of the most successful theories in all of the science extending its influence into cosmology. When cosmologists observed that the universe was actually expanding faster than before instead of slowing down owing to gravity, it seemed like some unknown energy was propelling this expansion. For lack of clarity on what this energy was, they named it **dark energy**. QFT explained dark energy with the virtual particles dwelling in the empty spaces.

The **Schrodinger equation** mainly addressed the wave function from a single frame of reference – the observer – while the **theory of relativity** specifically defined the frame of reference as a critical component of the space-time relationship. It seemed like even the last 100 years of quantum theory evolution was only scratching at the surface of an ever-expanding multiverse of possibilities. It was becoming increasingly apparent that there could exist a grander more unifying theory-- one that could explain both quantum theory and Einstein's theory of relativity. The ultimate goal thus became to find a theory that would reveal and explain the very fabric of reality: a Theory of Everything. *That search is still on.*
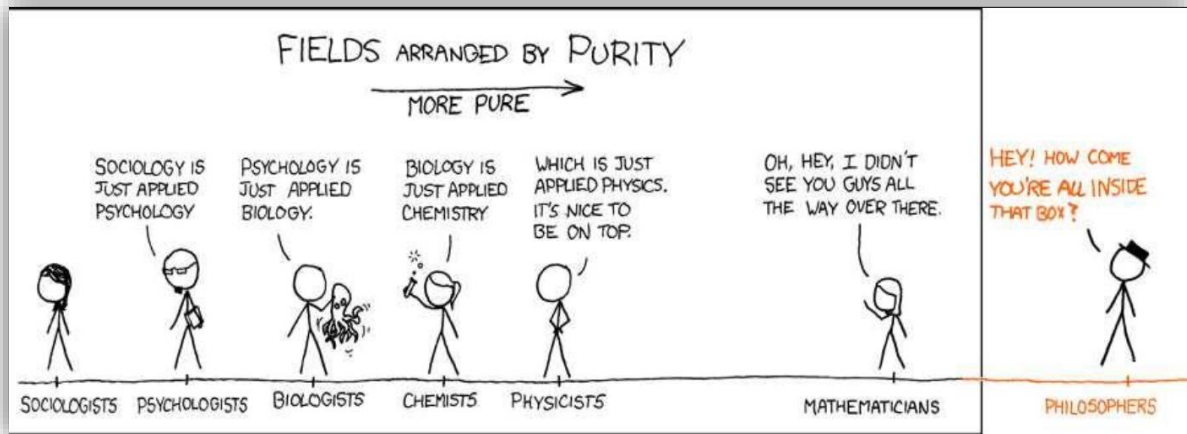
Figure 27: Fields of Purity. A Comic relief *(Image by https://xkcd.com/435/)*

If we succeed in building quantum computers with exponential speedup and massive scale, they would be ideal in realizing this theory of everything. Quantum machines are more natural than classical computers we have built. The universe itself can be considered as a quantum computer churning away infinite amounts of data, tracking hidden state and isolating systems in a coherent state, and possibly the very source code of our boundless reality.

There is a prevalent interpretation of quantum mechanics that dabbles in simulation theory – **The Simulation Hypothesis**[31]. According to this materialistic view, our universe is most likely a simulation in a physical universe. The futuristic evolution of technology leads to posthuman lifeforms that are capable of producing large quantities of high fidelity simulations, called ancestor simulations with the final deduction that it is more plausible that we exist purely in one of the numerous simulations rather than the real universe. Our reality, as we perceive it, is simply input data fed to these simulations.

Philosophy continues to influence science by essentially framing the questions while science sets the strategy to discover the consensus. As we progress, and progress we must; we need to prioritize uprooting us mere mortals from the protractedness of our anthropocentrism, which, if left uncontained, will result in the absolute and inevitable failure of our biome.

If human fortitude is any indicator, it's my firm belief that the coming century should be full of new and exciting quantum wonders. Unless, we indeed are in a simulation, and our posthuman overlords pull the trigger on us and *reboot reality.*

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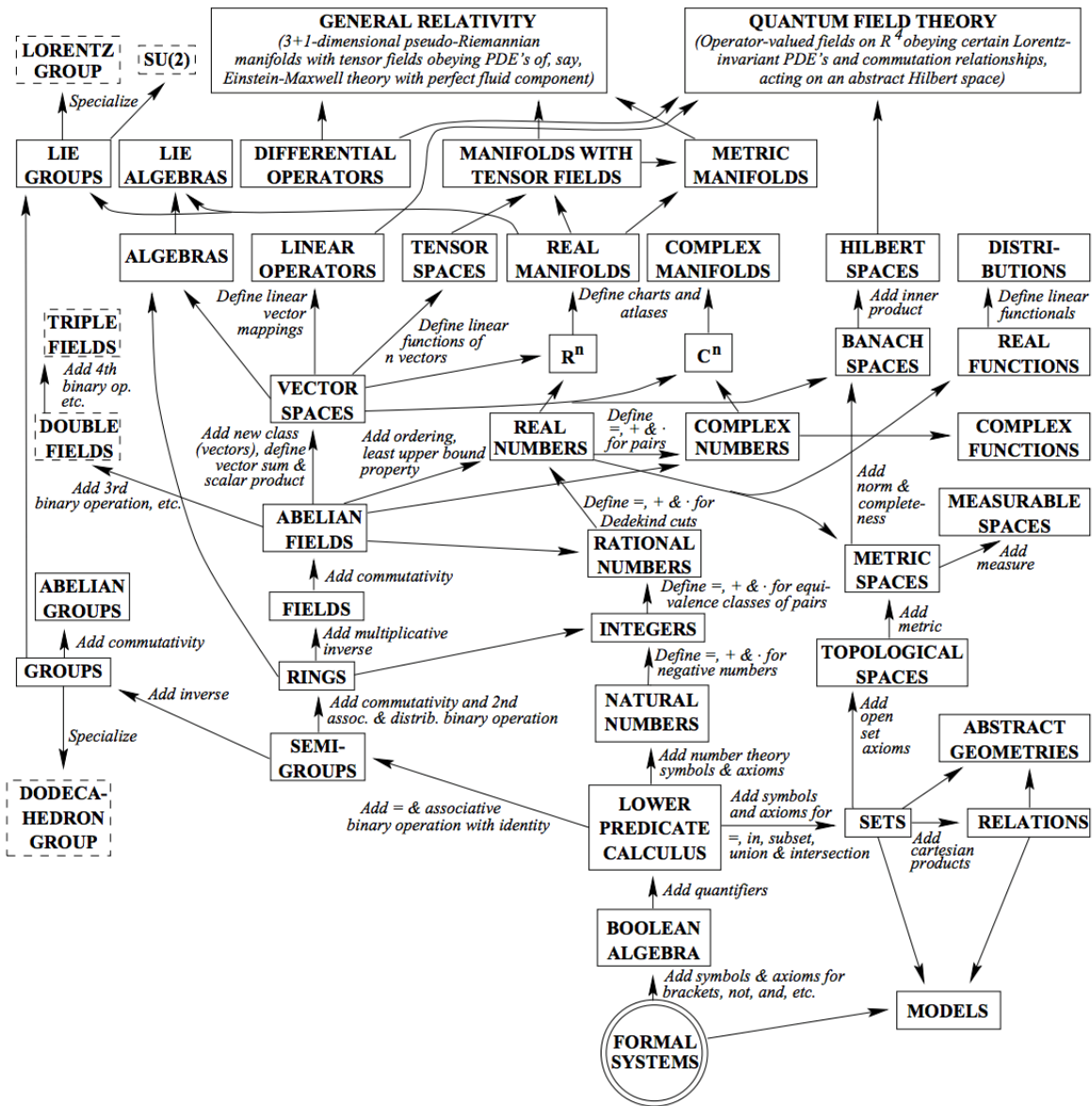**T.** Original Source: Definitions, Terms & Schema developed by Author

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Is "the theory of everything" merely the ultimate ensemble theory? By Max Tegmark [32]

LORENTZ GROUP | **SU(2)**

GENERAL RELATIVITY
(3+1-dimensional pseudo-Riemannian manifolds with tensor fields obeying PDE's of, say, Einstein-Maxwell theory with perfect fluid component)

QUANTUM FIELD THEORY
(Operator-valued fields on R^4 obeying certain Lorentz-invariant PDE's and commutation relationships, acting on an abstract Hilbert space)

Specialize

LIE GROUPS | **LIE ALGEBRAS** | **DIFFERENTIAL OPERATORS** | **MANIFOLDS WITH TENSOR FIELDS** | **METRIC MANIFOLDS**

ALGEBRAS | **LINEAR OPERATORS** | **TENSOR SPACES** | **REAL MANIFOLDS** | **COMPLEX MANIFOLDS** | **HILBERT SPACES** | **DISTRI-BUTIONS**

Define linear vector mappings
Define linear functions of n vectors
Define charts and atlases
Add inner product
Define linear functionals

TRIPLE FIELDS
Add 4th binary op. etc.

DOUBLE FIELDS
Add 3rd binary operation, etc.

VECTOR SPACES

R^n | C^n

BANACH SPACES | **REAL FUNCTIONS**

Add new class (vectors), define vector sum & scalar product
Add ordering, least upper bound property

REAL NUMBERS

Define =, + & · for pairs

COMPLEX NUMBERS

COMPLEX FUNCTIONS

Add norm & completeness

ABELIAN FIELDS

Define =, + & · for Dedekind cuts

RATIONAL NUMBERS

MEASURABLE SPACES

METRIC SPACES

Add measure

ABELIAN GROUPS

Add commutativity

FIELDS

Add multiplicative inverse

Define =, + & · for equivalence classes of pairs

INTEGERS

Add metric

TOPOLOGICAL SPACES

Add commutativity

GROUPS

Add inverse

RINGS

Add commutativity and 2nd assoc. & distrib. binary operation

Define =, + & · for negative numbers

NATURAL NUMBERS

Add open set axioms

ABSTRACT GEOMETRIES

Specialize

DODECA-HEDRON GROUP

SEMI-GROUPS

Add = & associative binary operation with identity

LOWER PREDICATE CALCULUS

Add number theory symbols & axioms

Add symbols and axioms for =, in, subset, union & intersection

SETS | **RELATIONS**

Add cartesian products

Add quantifiers

BOOLEAN ALGEBRA

Add symbols & axioms for brackets, not, and, etc.

FORMAL SYSTEMS

MODELS

Relationships between various basic mathematical structures. The arrows generally indicate addition of new symbols and/or axioms. Arrows that meet indicate the combination of structures — for instance, an algebra is a vector space that is also a ring, and a Lie group is a group that is also a manifold.